Transportation Models

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PowerPoint presentation to accompany
Heizer, Render, Munson
Operations Management, Twelfth Edition, Global Edition
Principles of Operations Management, Tenth Edition, Global Edition

PowerPoint slides by Jeff Heyl

Outline

- Transportation Modeling
- Developing an Initial Solution
- The Stepping-Stone Method
- Special Issues in Modeling

Learning Objectives

When you complete this chapter you should be able to:

C.1 ***Develop*** an initial solution to a transportation models with the northwest-corner and intuitive lowest-cost methods

C.2 ***Solve*** a problem with the stepping-stone method

C.3 ***Balance*** a transportation problem

C.4 ***Deal*** with a problem that has degeneracy

Transportation Modeling

- An iterative procedure that finds the least costly means of moving products from a series of sources to a series of destinations
- Can be used to help resolve distribution and location decisions

Transportation Modeling

- A special class of linear programming
- Need to know
  1. The *origin points* or *sources* and the capacity or supply per period at each
  2. The *destinations* and the demand per period at each
  3. The cost of shipping one unit from each origin to each destination

Transportation Problem

TABLE C.1	Transportation Costs per Bathtub for Arizona Plumbing		
TO / FROM	ALBUQUERQUE	BOSTON	CLEVELAND
Des Moines	$5	$4	$3
Evansville	$8	$4	$3
Fort Lauderdale	$9	$7	$5

Transportation Problem

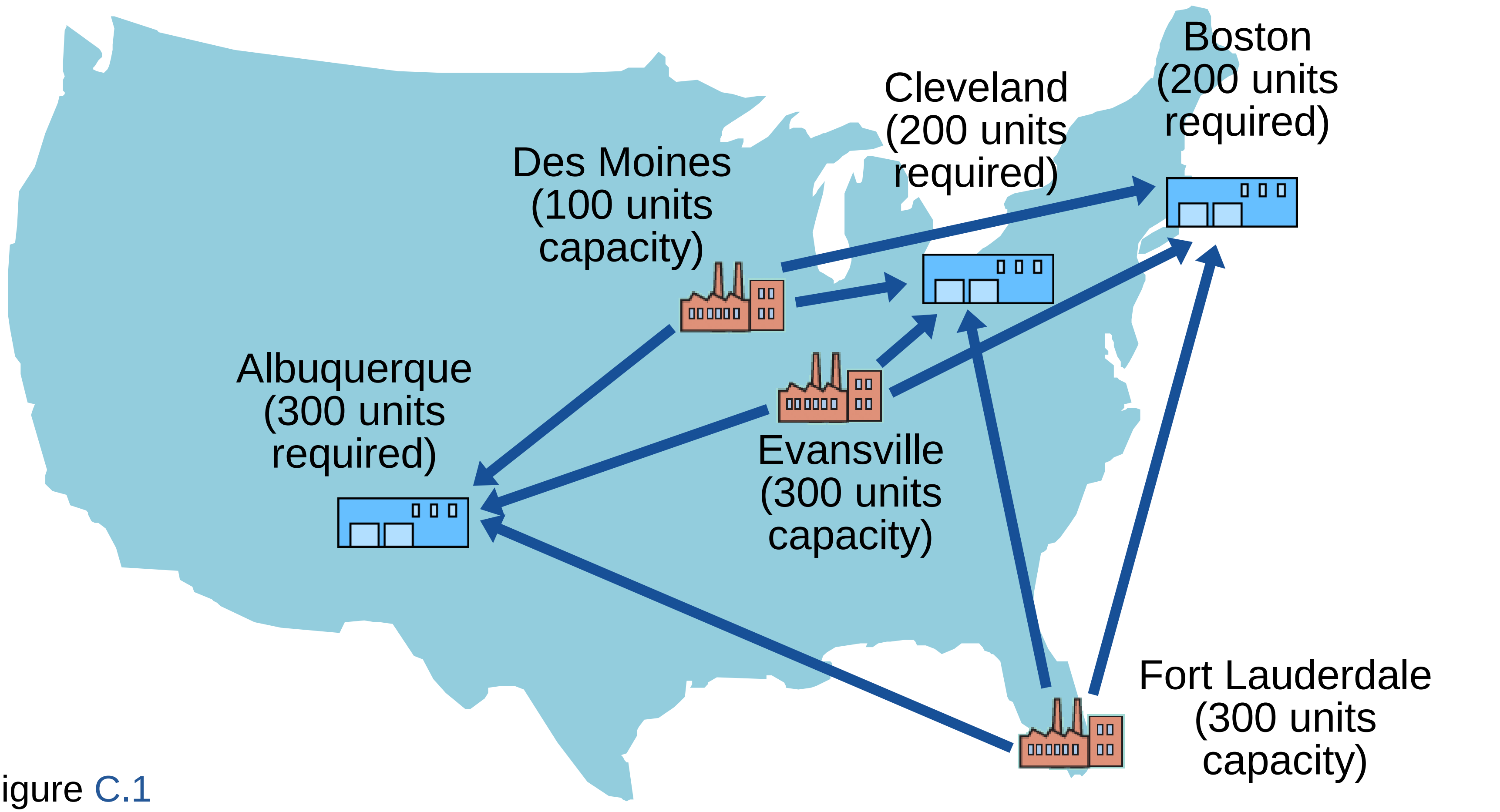

Figure C.1

Transportation Matrix

Figure C.2

From \ To	Albuquerque	Boston	Cleveland	Factory capacity
Des Moines	$5	$4	$3	100
Evansville	$8	$4	$3	300
Fort Lauderdale	$9	$7	$5	300
Warehouse requirement	300	200	200	700

Des Moines capacity constraint

Cell representing a possible source-to-destination shipping assignment (Evansville to Cleveland)

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Cleveland warehouse demand

Total demand and total supply

Northwest-Corner Rule

- Start in the upper left-hand cell (or northwest corner) of the table and allocate units to shipping routes as follows:
  1. Exhaust the supply (factory capacity) of each row before moving down to the next row
  2. Exhaust the (warehouse) requirements of each column before moving to the next column
  3. Check to ensure that all supplies and demands are met

Northwest-Corner Rule

1. Assign 100 tubs from Des Moines to Albuquerque (exhausting Des Moines' supply)
2. Assign 200 tubs from Evansville to Albuquerque (exhausting Albuquerque's demand)
3. Assign 100 tubs from Evansville to Boston (exhausting Evansville's supply)
4. Assign 100 tubs from Fort Lauderdale to Boston (exhausting Boston's demand)
5. Assign 200 tubs from Fort Lauderdale to Cleveland (exhausting Cleveland's demand and Fort Lauderdale's supply)

Northwest-Corner Rule

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 $5	$4	$3	100
(E) Evansville	200 $8	100 $4	$3	300
(F) Fort Lauderdale	$9	100 $7	200 $5	300
Warehouse requirement	300	200	200	700

Means that the firm is shipping 100 bathtubs from Fort Lauderdale to Boston

Figure C.3

Northwest-Corner Rule

TABLE C.2 Computed Shipping Cost

ROUTE				
FROM	TO	TUBS SHIPPED	COST PER UNIT	TOTAL COST
D	A	100	$5	$ 500
E	A	200	8	1,600
E	B	100	4	400
F	B	100	7	700
F	C	200	5	$1,000
				$4,200

This is a feasible solution but not necessarily the lowest-cost alternative

Intuitive Lowest-Cost Method

1. Identify the cell with the lowest cost
2. Allocate as many units as possible to that cell without exceeding supply or demand; then cross out the row or column (or both) that is exhausted by this assignment
3. Find the cell with the lowest cost from the remaining cells
4. Repeat steps 2 and 3 until all units have been allocated

Intuitive Lowest-Cost Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	$5	$4	$3 100	100
(E) Evansville	$8	$4	$3	300
(F) Fort Lauderdale	$9	$7	$5	300
Warehouse requirement	300	200	200	700

First, $3 is the lowest cost cell so ship 100 units from Des Moines to Cleveland and cross off the first row as Des Moines is satisfied

Figure C.4

Intuitive Lowest-Cost Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	$5	$4	$3 100	100
(E) Evansville	$8	$4	$3 100	300
(F) Fort Lauderdale	$9	$7	$5	300
Warehouse requirement	300	200	200	700

Second, $3 is again the lowest cost cell so ship 100 units from Evansville to Cleveland and cross off column C as Cleveland is satisfied

Figure C.4

Intuitive Lowest-Cost Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	$5	$4	$3 100	100
(E) Evansville	$8	$4 200	$3 100	300
(F) Fort Lauderdale	$9	$7	$5	300
Warehouse requirement	300	200	200	700

Third, $4 is the lowest cost cell so ship 200 units from Evansville to Boston and cross off column B and row E as Evansville and Boston are satisfied

Figure C.4

Intuitive Lowest-Cost Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	$5	$4	100 $3	100
(E) Evansville	$8	200 $4	100 $3	300
(F) Fort Lauderdale	300 $9	$7	$5	300
Warehouse requirement	300	200	200	700

Finally, ship 300 units from Albuquerque to Fort Lauderdale as this is the only remaining cell to complete the allocations

Figure C.4

Intuitive Lowest-Cost Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	$5	$4	100 $3	100
(E) Evansville	$8	200 $4	100 $3	300
(F) Fort Lauderdale	300 $9	$7	$5	300
Warehouse requirement	300	200	200	700

Total Cost = $3(100) + $3(100) + $4(200) + $9(300)
= $4,100

Figure C.4

Intuitive Lowest-Cost Method

This is a feasible solution, and an improvement over the previous solution, but not necessarily the lowest-cost alternative

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
[illegible]	[illegible]	$4	100 $3	100
[illegible]	[illegible]	$4	100 $3	300
[illegible]	[illegible]	$7	$5	300
Warehouse requirement	300	200	200	700

Total Cost = \$3(100) + \$3(100) + \$4(200) + \$9(300)
= \$4,100

Figure C.4

Stepping-Stone Method

1. Select any unused square to evaluate
2. Beginning at this square, trace a closed path back to the original square via squares that are currently being used
3. Beginning with a plus (+) sign at the unused square, place alternate minus and plus signs at each corner of the path just traced

Stepping-Stone Method

4. Calculate an improvement index by first adding the unit-cost figures found in each square containing a plus sign and subtracting the unit costs in each square containing a minus sign
5. Repeat steps 1 though 4 until you have calculated an improvement index for all unused squares. If all indices are ≥ 0, you have reached an optimal solution.

Stepping-Stone Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 $5	$4	$3	100
(E) Evansville	200 $8	100 $4	$3	300
(F) Fort Lauderdale	$9	100 $7	200 $5	300
Warehouse requirement	300	200	200	700

Des Moines-Boston index

$= \$4 - \$5 + \$8 - \$4$

$= +\$3$

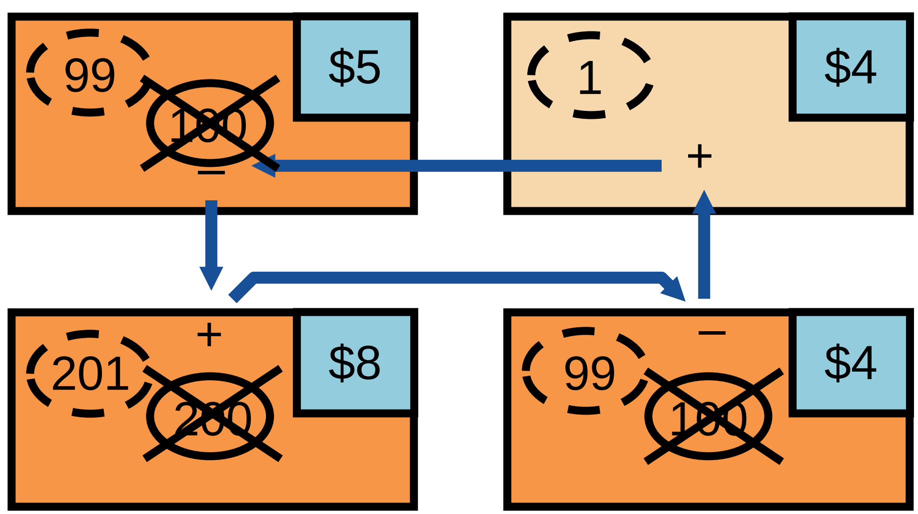

Figure C.5

Stepping-Stone Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	$5 100 –	$4	$3 Start +	100
(E) Evansville	$8 200 +	$4 100 –	$3	300
(F) Fort Lauderdale	$9	$7 100 +	$5 200 –	300
Warehouse requirement	300	200	200	700

Des Moines-Cleveland index

= $3 – $5 + $8 – $4 + $7 – $5 = +$4

Figure C.6

Stepping-Stone Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 \$5	\$4	\$3	100
(E) Evansville				
(F) Fort Laude				
Warehouse requirement				

Evansville-Cleveland index

= \$3 – \$4 + \$7 – \$5 = +\$1

(Closed path = EC – EB + FB – FC)

Fort Lauderdale-Albuquerque index

= \$9 – \$7 + \$4 – \$8 = –\$2

(Closed path = FA – FB + EB – EA)

Stepping-Stone Method

1. If an improvement is possible, choose the route (unused square) with the largest negative improvement index
2. On the closed path for that route, select the smallest number found in the squares containing minus signs
3. Add this number to all squares on the closed path with plus signs and subtract it from all squares with minus signs

Stepping-Stone Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 $5	$4	$3	100
(E) Evansville	200 $8 −	100 $4 +	$3	300
(F) Fort Lauderdale	$9 +	100 $7 −	200 $5	300
Warehouse requirement				

1. Add 100 units on route FA
2. Subtract 100 from route FB
3. Add 100 to route EB
4. Subtract 100 from route EA

Figure C.7

Stepping-Stone Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 ($5)	($4)	($3)	100
(E) Evansville	100 ($8)	200 ($4)	($3)	300
(F) Fort Lauderdale	100 ($9)	($7)	200 ($5)	300
Warehouse requirement	300	200	200	700

Total Cost = $5(100) + $8(100) + $4(200) + $9(100) + $5(200)
= $4,000

Figure C.8

Special Issues in Modeling

- Demand not equal to supply
  - Called an unbalanced problem
  - Common situation in the real world
  - Resolved by introducing *dummy sources* or *dummy destinations* as necessary with cost coefficients of zero

Special Issues in Modeling

Total Cost = 250($5) + 50($8) + 200($4) + 50($3) + 150($5) + 150(0)
= $3,350

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Dummy	Factory capacity
(D) Des Moines	250 ($5)	($4)	($3)	(0)	250
(E) Evansville	50 ($8)	200 ($4)	50 ($3)	(0)	300
(F) Fort Lauderdale	($9)	($7)	150 ($5)	150 (0)	300
Warehouse requirement	300	200	200	150	850

New Des Moines capacity

Special Issues in Modeling

- Degeneracy
  - To use the stepping-stone methodology, *the number of occupied squares in any solution (initial or later) must be equal to the number of rows in the table plus the number of columns minus 1*
  - If a solution does not satisfy this rule it is called *degenerate*

Special Issues in Modeling

From \ To	Customer 1	Customer 2	Customer 3	Warehouse supply
Warehouse 1	100 $8	$2	$6	100
Warehouse 2	0 $10	100 $9	20 $9	120
Warehouse 3	$7	$10	80 $7	80
Customer demand	100	100	100	300

Initial solution is degenerate

Place a zero or *very* small quantity in an unused square and proceed computing improvement indices