Inventory Management (II)

12

PowerPoint presentation to accompany
Heizer, Render, Munson
Operations Management, Twelfth Edition, Global Edition
Principles of Operations Management, Tenth Edition, Global Edition
PowerPoint slides by Jeff Heyl

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Record Accuracy

- Accurate records are a critical ingredient in production and inventory systems
  - *Periodic systems* require regular checks of inventory
    - *Two-bin system*
  - *Perpetual inventory* tracks receipts and subtractions on a continuing basis
    - May be semi-automated

Record Accuracy

- Incoming and outgoing record keeping must be accurate

- Stockrooms should be secure
- Necessary to make precise decisions about ordering, scheduling, and shipping

Control of Service Inventories

- Can be a critical component of profitability
- Losses may come from shrinkage or pilferage
- Applicable techniques include
  1. *Proper personnel selection, training and discipline*
  2. *Tight control of incoming shipments*
  3. *Effective control of all goods leaving facility*

Holding Costs

TABLE 12.1	Determining Inventory Holding Costs
CATEGORY	**COST (AND RANGE) AS A PERCENT OF INVENTORY VALUE**
Housing costs (building rent or depreciation, operating costs, taxes, insurance)	6% (3 - 10%)
Material handling costs (equipment lease or depreciation, power, operating cost)	3% (1 - 3.5%)
Labor cost (receiving, warehousing, security)	3% (3 - 5%)
Investment costs (borrowing costs, taxes, and insurance on inventory)	11% (6 - 24%)
Pilferage, space, and obsolescence (much higher in industries undergoing rapid change like tablets and smart phones)	3% (2 - 5%)
Overall carrying cost	26%

Holding Costs

TABLE 12.1	Determining Inventory Holding Costs
CA[illegible]	COS[illegible]
I[illegible] c[illegible]	
M[illegible] d[illegible]	
L[illegible]	[illegible]%)
In[illegible] (…rowing costs, taxes, and insurance on inventory)	11% (6 - 24%)
Pilferage, space, and obsolescence (much higher in industries undergoing rapid change like PCs and cell phones)	3% (2 - 5%)
Overall carrying cost	26%

Holding costs vary considerably depending on the business, location, and interest rates. Generally greater than 15%, some high tech and fashion items have holding costs greater than 40%.

Inventory Models for Independent Demand

Need to determine when and how much to order

1. Basic economic order quantity (EOQ) model
2. Quantity discount model
3. Production order quantity model

EOQ is a Robust Model

- The EOQ model is **robust**
- It works even if all parameters and assumptions are not met
- The total cost curve is relatively flat in the area of the EOQ

An EOQ Example

Determine optimal number of needles to order

D = ~~1,000 units~~ 1,500 units

S = $10 per order

H = $.50 per unit per year

N= 5 orders/year

$Q^*_{1,000}$ = 200 units

T = 50 days

$Q^*_{1,500}$ = 244.9 units

Ordering previous *Q**

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$= \frac{1,500}{200}(\$10) + \frac{200}{2}(\$.50)$$

$$= \$75 + \$50 = \$125$$

Ordering new *Q**

$$= \frac{1,500}{244.9}(\$10) + \frac{244.9}{2}(\$.50)$$

$$= 6.125(\$10) + 122.45(\$.50)$$

$$= \$61.25 + \$61.22 = \$122.47$$

An EOQ Example

Determine optimal number of

D = ~~1,000 units~~ 1,500 units

S = $10 per order

H = $.50 per unit per year

N= 5 orders/year

> Only 2% less than the total cost of $125 when the order quantity was 200

Ordering old Q*

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$= \frac{1{,}500}{200}(\$10) + \frac{200}{2}(\$.50)$$

$$= \$75 + \$50 = \$125$$

Or

$$= \frac{1{,}500}{244.9}(\$10) + \frac{244.9}{2}(\$.50)$$

$$= 6.125(\$10) + 122.45(\$.50)$$

$$= \$61.25 + \$61.22 = \$122.47$$

Quantity Discount Models

- Reduced prices are often available when larger quantities are purchased
- Trade-off is between reduced purchasing (product) cost and increased holding cost

Example: Chris Beehner Electronics stocks toy remote control flying drones. Recently, the store has been offered a quantity discount schedule for these drones. This quantity schedule was shown in Table 12.2 . Furthermore, setup cost is $200 per order, annual demand is 5,200 units, and annual inventory carrying charge as a percent of cost, *I* , is 28%. What order quantity will minimize the total inventory cost?

TABLE 12.2 **A Quantity Discount Schedule**

PRICE RANGE	QUANTITY ORDERED	PRICE PER UNIT *P*
Initial price	0 to 119	$100
Discount price 1	200 to 1,499	$ 98
Discount price 2	1,500 and over	$ 96

Quantity Discount Models

Total annual cost = Setup cost + Holding cost + Product cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}IP + PD$$

where

Q = Quantity ordered
D = Annual demand in units
S = Ordering or setup cost per order
P = Price per unit
I = Holding cost per unit per year expressed as a percent of price P

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

Because unit price varies, holding cost is expressed as a percent (I) of unit price (P)

Quantity Discount Models

Steps in analyzing a quantity discount

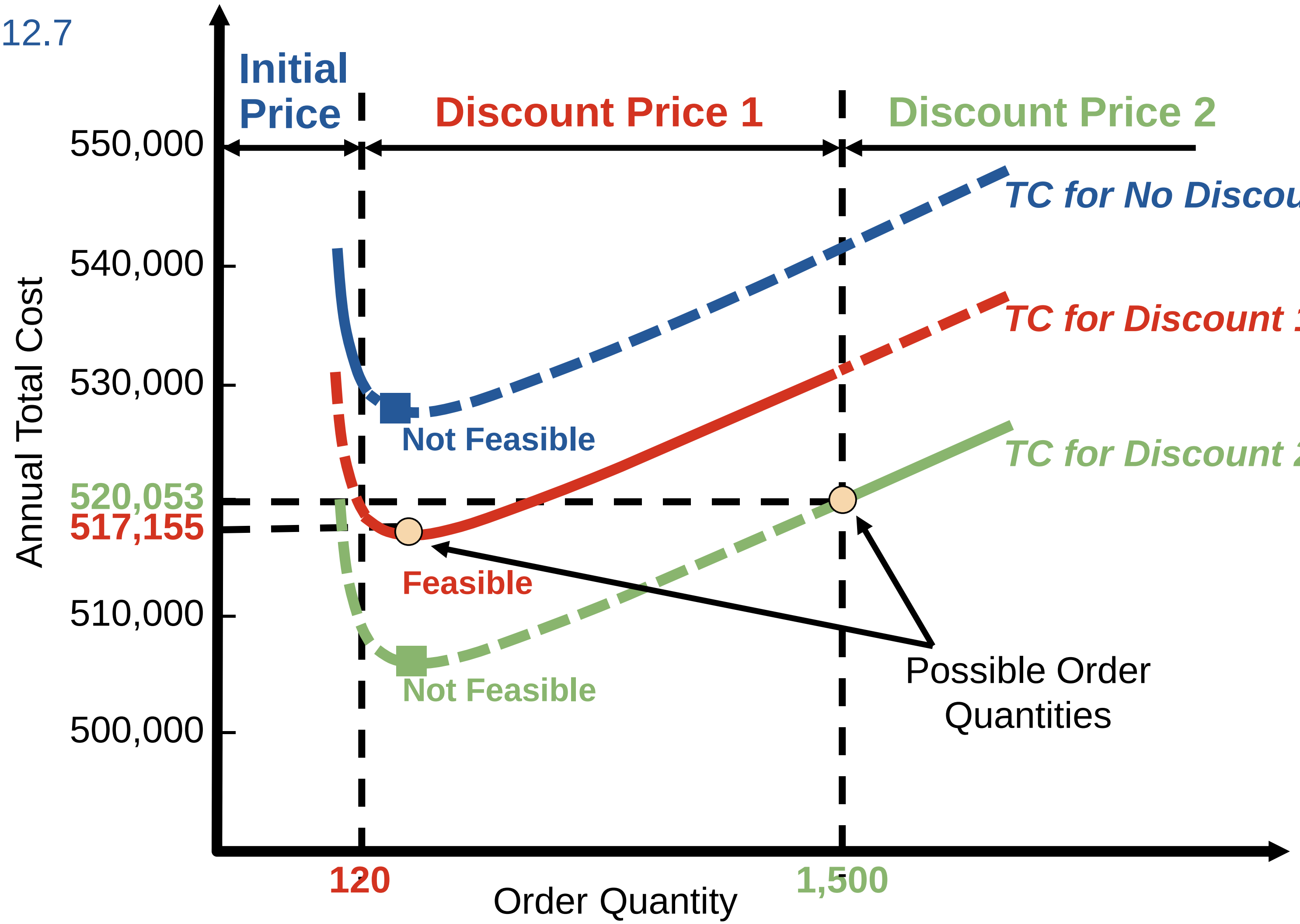
1. Starting with the *lowest* possible purchase price, calculate Q^* until the first feasible EOQ is found. This is a possible best order quantity, along with all price-break quantities for all *lower* prices.
2. Calculate the total annual cost for each possible order quantity determined in Step 1. Select the quantity that gives the lowest total cost.

Quantity Discount Models

- Recall that in the basic EOQ model, the determination of the optimal order size does not involve the purchasing cost, since under the assumption of no quantity discounts, price per unit is the same for all order quantities.
- When quantity discounts are offered, there is a separate U-shaped total-cost curve for each unit price.
- Because the unit prices are different, each curve is raised by a different amount: a smaller unit price will raise the total cost curve less than a larger unit price.
- No single curve applies to the entire range of quantities.
- Each total cost curve has its local minimum.

Quantity Discount Models

Figure 12.7



Quantity Discount Example

Calculate Q^* for every discount starting with the lowest price

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_{\$96}^* = \sqrt{\frac{2(5{,}200)(\$200)}{(.28)(\$96)}} = 278 \text{ drones/order}$$

Infeasible – calculate Q* for next-higher price

$$Q_{\$98}^* = \sqrt{\frac{2(5{,}200)(\$200)}{(.28)(\$98)}} = 275 \text{ drones/order}$$

Feasible

Quantity Discount Example

TABLE 12.3	Total Cost Computations for Chris Beehner Electronics				
ORDER QUANTITY	**UNIT PRICE**	**ANNUAL ORDERING COST**	**ANNUAL HOLDING COST**	**ANNUAL PRODUCT COST**	**TOTAL ANNUAL COST**
275	$98	$3,782	$3,773	$509,600	$517,155
1,500	$96	$693	$20,160	$499,200	$520,053

Choose the price and quantity that gives the lowest total cost

Buy 275 drones at $98 per unit

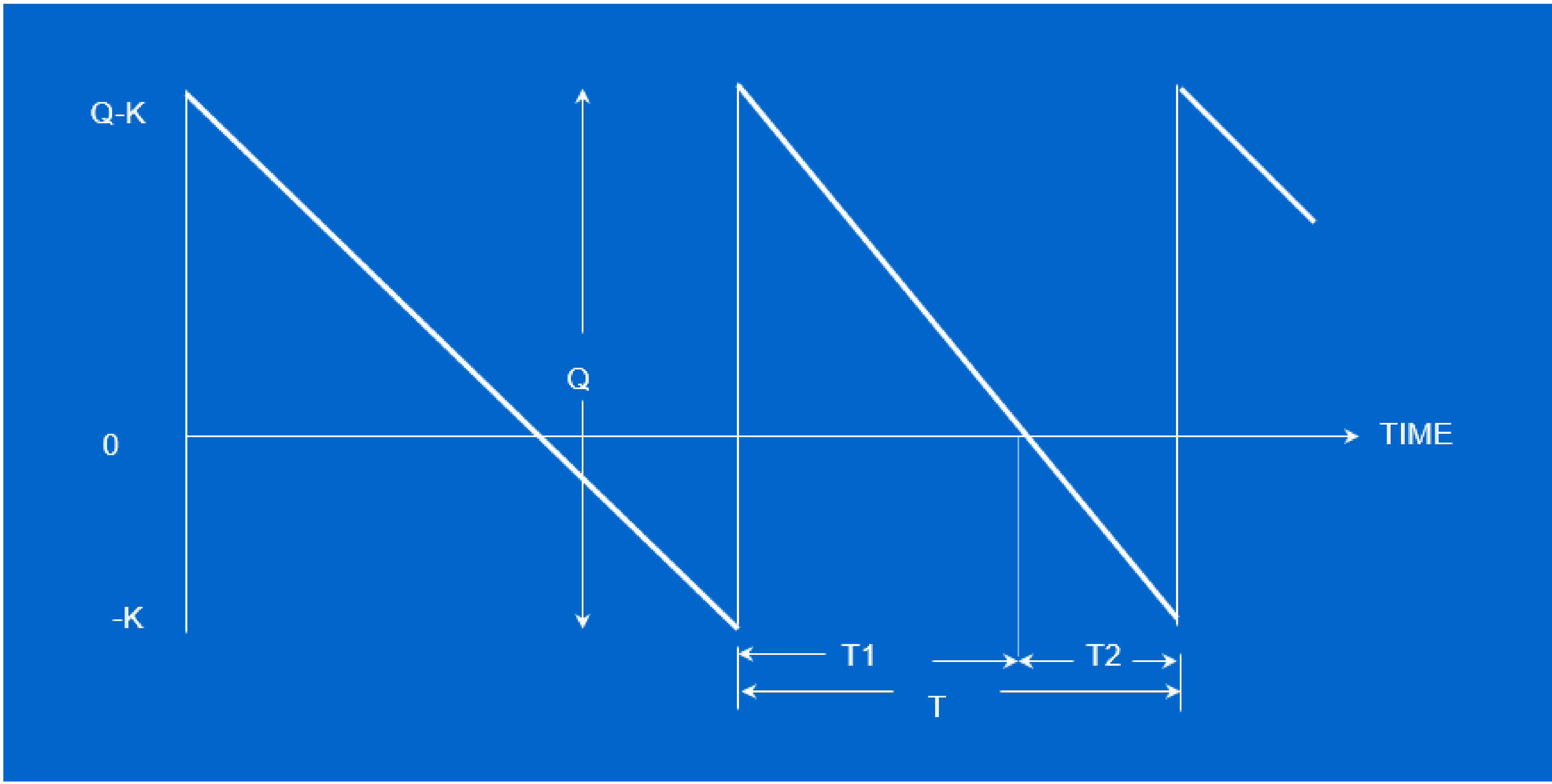
Quantity Discount Variations

- *All-units discount* is the most popular form
- *Incremental quantity discounts* apply only to those units purchased beyond the price break quantity
- *Fixed fees* may encourage larger purchases
- *Aggregation* over items or time
- *Truckload discounts*, *buy-one-get-one-free offers*, *one-time-only sales.*

EOQ with Planned Shortages

- Here the assumption that changes - in relation to the assumptions of the EOQ model - is that shortages are permissible to a certain extent. Demand is manifested when there is no stock and is met with delay when the new inventory batch (backordering) arrives. For each unit of product requested and not delivered directly due to shortage a cost of shortage B per unit of product and time unit is implied.
- The system's decision variables are the order quantity Q and the maximum allowable level of K (Stock out). The time evolution of the stock is illustrated in the next figure (negative inventory means pending demand).

EOQ with Planned Shortages



EOQ For Planned Shortages

- When the inventory level reaches – K then quantity Q arrives covering the demanded units and the inventory level reaches Q-K. The cycle time between two consecutive arrivals T is divided into two intervals: T1 (immediate demand coverage) and T2 (pending demand)
- The total cost function that must be minimized is:

$$TC_b = S\frac{D}{Q} + H\frac{(Q-K)^2}{2Q} + B\frac{K^2}{2Q}$$

EOQ for Planned Shortages

- The optimal order quantity that minimizes the total cost is:

$$Q^* = \sqrt{\frac{2DS}{H}\left(\frac{H+B}{B}\right)}$$

- The maximum level of unsatisfied demand is:

$$K^* = Q^*\left(\frac{H}{H+B}\right)$$

EOQ For Planned Shortages

- The time interval that the inventory is positive is:

$$T_1 = \frac{Q - K}{d}$$

where d is daily demand

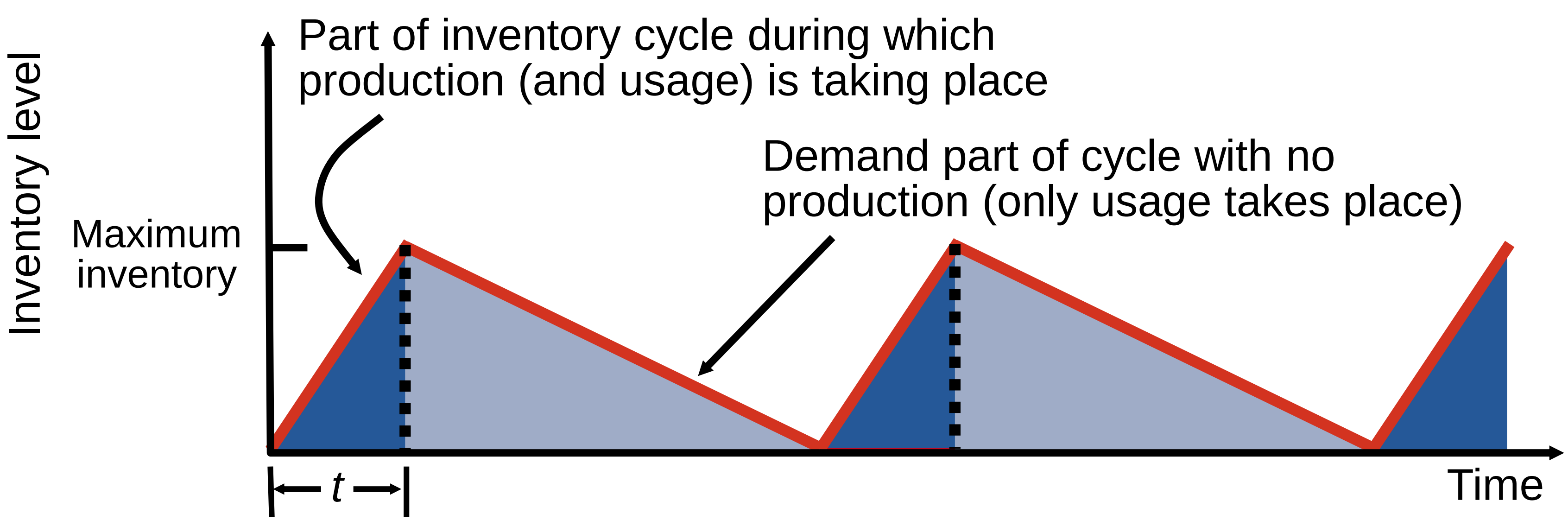
- The time interval that the inventory is negative is:

$$T_2 = \frac{K}{d}$$

Production Order Quantity Model

1. Used when inventory builds up over a period of time after an order is placed
2. Used when units are produced and sold simultaneously

Figure 12.6



Assumptions

- Only one item is involved
- Annual demand is known
- Usage rate is constant
- Usage occurs continually, but production occurs periodically
- Production rate is constant and Q is always the same
- Lead time is not zero but does not vary (including elements that correspond to the idle production times for machine calibration, tool placement, study of the data for the production of the new batch, removal of the previous production batch parts, cleaning of the production area, etc.)
- No quantity discounts

Production Order Quantity Model

Q = Number of units per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\begin{pmatrix}\text{Annual inventory} \\ \text{holding cost}\end{pmatrix} = (\text{Average inventory level}) \times \begin{pmatrix}\text{Holding cost} \\ \text{per unit per year}\end{pmatrix}$$

$$\begin{pmatrix}\text{Annual inventory} \\ \text{level}\end{pmatrix} = (\text{Maximum inventory level})/2$$

$$\begin{pmatrix}\text{Maximum} \\ \text{inventory level}\end{pmatrix} = \begin{pmatrix}\text{Total produced during} \\ \text{the production run}\end{pmatrix} - \begin{pmatrix}\text{Total used during} \\ \text{the production run}\end{pmatrix}$$

$$= pt - dt$$

Production Order Quantity Model

Q = Number of units per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\begin{pmatrix}\text{Maximum}\\\text{inventory level}\end{pmatrix} = \begin{pmatrix}\text{Total produced during}\\\text{the production run}\end{pmatrix} - \begin{pmatrix}\text{Total used during}\\\text{the production run}\end{pmatrix}$$

$$= pt - dt$$

However, Q = total produced = pt ; thus $t = Q/p$

$$\begin{pmatrix}\text{Maximum}\\\text{inventory level}\end{pmatrix} = p\left(\frac{Q}{p}\right) - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right)$$

$$\text{Holding cost} = \frac{\text{Maximum inventory level}}{2}(H) = \frac{Q}{2}\left[1 - \left(\frac{d}{p}\right)H\right]$$

Production Order Quantity Model

Q = Number of units per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\text{Setup cost} = (D/Q)S$$

$$\text{Holding cost} = \tfrac{1}{2}HQ\left[1 - \left(d/p\right)\right]$$

$$\frac{D}{Q}S = \tfrac{1}{2}HQ\left[1 - \left(d/p\right)\right]$$

$$Q^2 = \frac{2DS}{H\left[1 - \left(d/p\right)\right]}$$

$$Q_p^* = \sqrt{\frac{2DS}{H\left[1 - \left(d/p\right)\right]}}$$

Production Order Quantity Example

D = 1,000 units
S = $10
H = $0.50 per unit per year

p = 8 units per day
d = 4 units per day

$$Q_p^* = \sqrt{\frac{2DS}{H\left[1-\left(d/p\right)\right]}}$$

$$Q_p^* = \sqrt{\frac{2(1{,}000)(10)}{0.50\left[1-(4/8)\right]}}$$

$$= \sqrt{\frac{20{,}000}{0.50(1/2)}} = \sqrt{80{,}000}$$

$$= 282.8\text{, or } 283$$

Production Order Quantity Example

Note:

$$d = 4 = \frac{D}{\text{Number of days the plant is in operation}} = \frac{1{,}000}{250}$$

When annual data are used the equation becomes:

$$Q_p^* = \sqrt{\frac{2DS}{H\left(1 - \dfrac{\text{Annual demand rate}}{\text{Annual production rate}}\right)}}$$

Production Order Quantity Example II

A toy manufacturer uses 48,000 rubber wheels per year for its popular dump truck series. The firm makes its own wheels, which can produce at a rate of 800 per day. The toy trucks are assembled uniformly over the entire year. Carrying cost is $1 per wheel a year. Setup cost for a production run of wheels is $45. The firm operates 240 days per year. Determine the

a. Optimal run size

b. Minimum total annual cost for carrying and setup

c. Run time

Solution

D = 48000 wheels per year

S = $45

H = $1 per wheel per year

P = 800 wheels per day

U = 48000 wheels per 240 days, or 200 wheels per day

$$Q^* = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}} = \sqrt{\frac{2(48000)45}{1}}\sqrt{\frac{800}{800-200}} = 2400$$

Solution

$$I_{max} = \frac{Q^*}{p}(p-d) = \frac{2400}{800}(800-200) = 1800$$

$$TC = \frac{1800}{2} \times \$1 + \frac{48000}{2400} \times \$45 = \$900 + \$900 = \$1800$$

$$\text{cycle} = \frac{Q^*}{d} = \frac{2400}{200} = 12\text{days}$$

$$\text{Run} = \frac{Q^*}{p} = \frac{2400}{800} = 3\text{days}$$

Άσκηση 3 (Παππής, 2006, σελ. 140)

Μία μικρή εταιρεία παράγει μία ηλεκτρονική συσκευή που χρησιμοποιείται σε συστήματα πλοήγησης αεροσκαφών. Η ζήτηση βασίζεται σε παραγγελίες μέσω συμβολαίων για αεροσκάφη και είναι γνωστό ότι είναι ομοιόμορφη και με ρυθμό 6,400 μονάδες ετησίως. Ένα βολτόμετρο που χρησιμοποιείται σε αυτήν την συσκευή μπορεί να παραχθεί από την εταιρεία με ρυθμό 128 μονάδες ημερησίως. Το έτος έχει 250 εργάσιμες ημέρες. Το κόστος ετοιμασίας κάθε παρτίδας παραγωγής είναι 24 ευρώ, και το κόστος αποθήκευσης είναι 3 ευρώ ανά μονάδα προϊόντος ετησίως. Ζητείται η βέλτιστη πολιτική αποθεματοποίησης για αυτό το προϊόν.

Άσκηση 3

Λύση: Το βέλτιστο μέγεθος της παρτίδας παραγωγής είναι

$Q^{*}= [2DS/H(1-d/p)]^{1/2}=[2x24x6,400/3x(1-6,400/(128x250))]^{1/2}=358$ μονάδες.

Ο χρόνος τοποθέτησης κάθε παρτίδας παραγωγής είναι

$Cycle=Q^{*}/d=358/6,400=0.056$ χρόνια$=0.056x250=14$ εργάσιμες ημέρες.

Ο χρόνος που απαιτείται για την παραγωγή μίας παρτίδας είναι
$Run= Q^{*}/p=358/128=2.8$ εργάσιμες ημέρες.

Το ελάχιστο κόστος που αντιστοιχεί σε αυτό το μέγεθος παρτίδας είναι

$C_{total}^{min}=[2SHD(1-d/p)]^{1/2}=[2x24x3x6,400(1-6400/(128x250))]^{1/2}=858.65$ ευρώ/χρόνο.

Άσκηση 4 (Παππής 2006, σελ. 141)

Υποθέστε ότι η εταιρεία στην προηγούμενη άσκηση διακόπτει την παραγωγή βολτομέτρων και τα παραγγέλλει από έναν εξωτερικό προμηθευτή με κόστος 20 ευρώ ανά μονάδα προϊόντος. Το κόστος έλλειψης είναι 40 ευρώ ανά μονάδα προϊόντος ετησίως εξαιτίας της επιπρόσθετης εργασίας που απαιτείται για την αποσυναρμολόγηση των μερικώς ολοκληρωμένων συσκευών. Το κόστος παραγγελίας είναι 27 ευρώ, και το κόστος αποθήκευσης υπολογίζεται στο 30% του επενδυμένου κεφαλαίου. Η διοίκηση επιθυμεί να γνωρίζει τις προκύπτουσες αλλαγές στην πολιτική αναπλήρωσης του αποθέματος των βολτομέτρων.

Άσκηση 4

Λύση: Η βέλτιστη ποσότητα παραγγελίας είναι η

$Q^{*}=(2SD/H)^{1/2}[(B+H)/B]^{1/2}=(2x27x6,400/0.3x20)^{1/2}((40+0.3x20)/(40))^{1/2}=258$ μονάδες

Το μέγιστο επίπεδο αποθέματος είναι

$I_{max}=Q^{*}-K^{*}=(2SD/H)^{1/2}[B/(B+H)]^{1/2}=(2x27x6,400/0.3x20)^{1/2}(40/(40+0.3x20))^{1/2}=$ 224 μονάδες.

Ο χρόνος τοποθέτησης αναπαραγγελίας είναι

$T=Q^{*}/d=258/6,400=0.04$ χρόνια$=0.04x250=10$ εργάσιμες ημέρες.

Η ποσότητα της ζήτησης που δεν καλύπτεται σε κάθε κύκλο ισούται με $K^{*}=Q^{*}-I_{max}=258-224=34$ μονάδες.