DEFINITION 9.5 P-Value

To obtain the ***P-value*** of a hypothesis test, we assume that the null hypothesis is true and compute the probability of observing a value of the test statistic as extreme or more extreme than that observed. By *extreme* we mean "far from what we would expect to observe if the null hypothesis is true." We use the letter $\boldsymbol{P}$ to denote the P-value.

What Does it Mean?

Small *P*-values provide evidence against the null hypothesis; larger *P*-values do not. The smaller (closer to 0) the *P*-value, the stronger the evidence is against the null hypothesis.

OBTAINING *P*-VALUES FOR A ONE-SAMPLE *z*-TEST

Recall that the test statistic for a one-sample *z*-test for a population mean with null hypothesis H_0: $\mu = \mu_0$ is

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}.$$

FIGURE 9.13
P-value for a z-test if the test is (a) two-tailed, (b) left-tailed, or (c) right-tailed

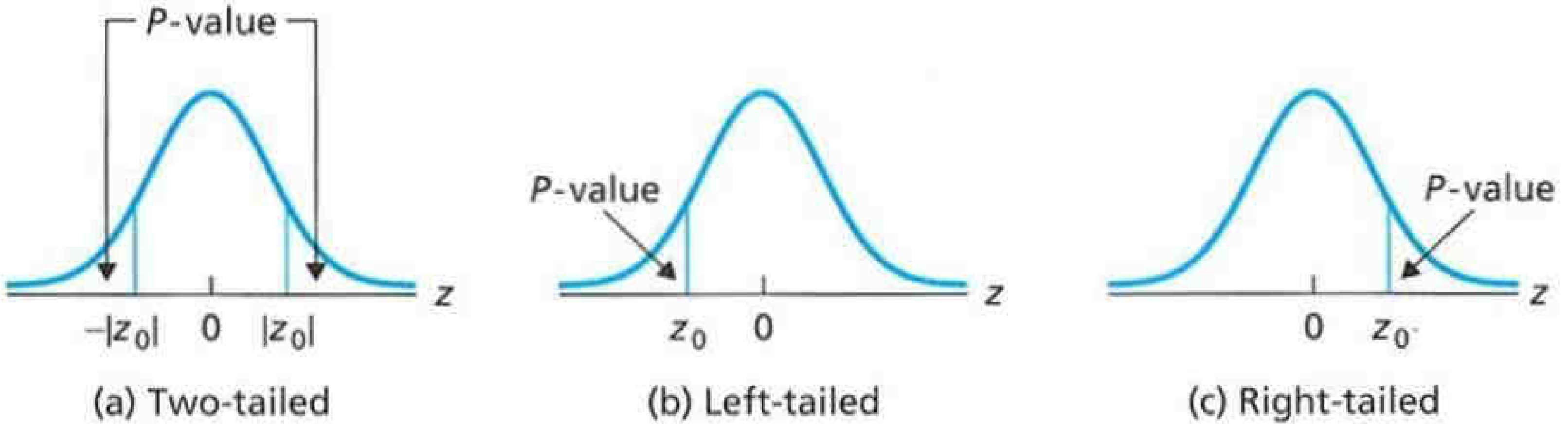


TABLE 9.5

This year's prices ($) for 40 history books

48.04	38.29	39.38	46.03
45.75	39.92	46.86	47.77
43.04	53.74	39.07	54.72
39.84	42.93	44.40	42.99
43.32	42.98	52.74	64.42
39.74	48.20	44.37	43.74
45.84	42.94	55.78	44.91
33.12	56.97	49.48	46.13
67.41	48.52	61.08	34.75
45.80	64.21	53.30	34.69

Descriptive Statistics

	PRICE
N	40
Mean	46.930
SD	8.0931
SE Mean	1.2796
Minimum	33.120
Maximum	67.410

Step 1 **State the null and alternative hypotheses.**

Let μ denote this year's mean retail price of all history books. We stated the null and alternative hypotheses in Example 9.2 as

$$H_0: \mu = \$43.50 \text{ (mean price has not increased)}$$
$$H_a: \mu > \$43.50 \text{ (mean price has increased).}$$

Note that the hypothesis test is right-tailed because a greater-than sign (>) appears in the alternative hypothesis.

Step 3 **Compute the value of the test statistic**

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}.$$

We have $\mu_0 = 43.50$, $\sigma = 7.61$, and $n = 40$. The mean of the sample data in Table 9.5 is $\bar{x} = 46.93$. Thus the value of the test statistic is

$$z = \frac{46.93 - 43.50}{7.61/\sqrt{40}} = 2.85.$$

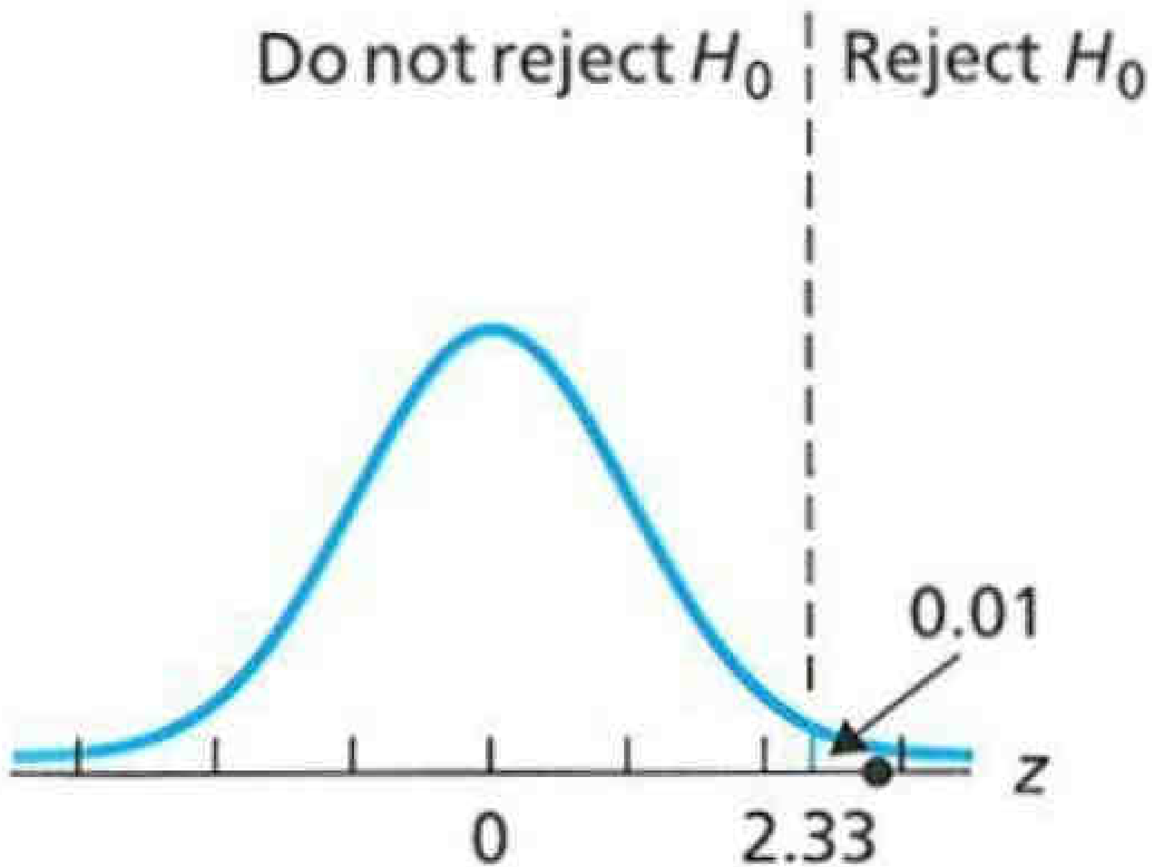
This value of z is marked with a dot in Fig. 9.9.

Step 4 The critical value for a right-tailed test is z_α.

As $\alpha = 0.01$, the critical value is $z_{0.01}$. From Table II (or Table 9.4 on page 388), $z_{0.01} = 2.33$, as shown in Fig. 9.9.

FIGURE 9.9

Criterion for deciding whether to reject the null hypothesis



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

The value of the test statistic, found in Step 3, is $z = 2.85$. Figure 9.9 reveals that this value falls in the rejection region, so we reject H_0. The test results are statistically significant at the 1% level.

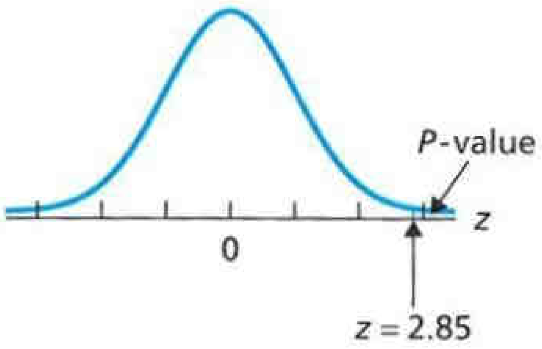
Step 6 Interpret the results of the hypothesis test.

What Does it Mean?

At the 1% significance level, the data provide sufficient evidence to conclude that this year's mean retail price of all history books has increased from the 1997 mean of $43.50.

Solution Because the hypothesis test is a right-tailed z-test, the P-value is the probability of observing a value of z of 2.85 or greater if the null hypothesis is true. That probability equals the area under the standard normal curve to the right of 2.85, the shaded area shown in Fig. 9.14. From Table II, we find that area to be $1 - 0.9978 = 0.0022$.

FIGURE 9.14
P-value for the history book hypothesis test



Example 9.9 *The One-Sample z-Test*

Clocking the Cheetah The Cheetah (*Acinonyx jubatus*) is the fastest land mammal on earth and is highly specialized to run down prey. According to the Cheetah Conservation of Southern Africa *Trade Environment Database*, the cheetah often exceeds speeds of 60 miles per hour (mph) and has been clocked at speeds of more than 70 mph.

One common estimate of mean top speed for cheetahs is 60 mph. Table 9.7 gives the top speeds, in miles per hour, over a quarter mile for a sample of 35 cheetahs. Do the data provide sufficient evidence to conclude that the mean top speed of all cheetahs differs from 60 mph? Assume that the population standard deviation of top speeds is 3.2 mph.

TABLE 9.7
Top speeds, in miles per hour, for a sample of 35 cheetahs

57.3	57.5	59.0	56.5	61.3	57.6	59.2
65.0	60.1	59.7	62.6	52.6	60.7	62.3
65.2	54.8	55.4	55.5	57.8	58.7	57.8
60.9	75.3	60.6	58.1	55.9	61.6	59.6
59.8	63.4	54.7	60.2	52.4	58.3	66.0

Example 9.11 *P*-Values

Clocking the Cheetah In Example 9.9, we conducted a hypothesis test to decide whether the mean top speed of all cheetahs differs from 60 mph. Recall that the null and alternative hypotheses are

$$H_0\text{: } \mu = 60 \text{ mph (mean top speed of cheetahs is 60 mph)}$$

$$H_a\text{: } \mu \neq 60 \text{ mph (mean top speed of cheetahs is not 60 mph),}$$

where μ denotes the mean top speed of all cheetahs. Note that the hypothesis test is two-tailed because a does-not-equal sign ($\neq$) appears in the alternative hypothesis.

Step 3 **Compute the value of the test statistic**

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}.$$

We have $\mu_0 = 60$, $\sigma = 3.2$, and $n = 35$. From the data in Table 9.7, we find that $\bar{x} = 59.526$. Thus the value of the test statistic is

$$z = \frac{59.526 - 60}{3.2/\sqrt{35}} = -0.88,$$

This value of z is marked with a solid dot in Fig. 9.12.

What Does it Mean?

The data provide very strong evidence against the null hypothesis.

Therefore the P-value of this hypothesis test is 0.0022. If the null hypothesis is true, we would observe a value of the test statistic z of 2.85 or greater only about 2 times in 1000. In other words, if the null hypothesis is true, a random sample of 40 history books would have a mean of \$46.93, or greater, about 0.2% of the time. ◆

Procedure 9.2

The One-Sample z-Test for a Population Mean (P-Value Approach)

Assumptions

1. Normal population or large sample
2. σ known

Step 1 The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is

$$H_a: \mu \neq \mu_0 \text{ (Two-tailed)} \quad \text{or} \quad H_a: \mu < \mu_0 \text{ (Left-tailed)} \quad \text{or} \quad H_a: \mu > \mu_0 \text{ (Right-tailed)}$$

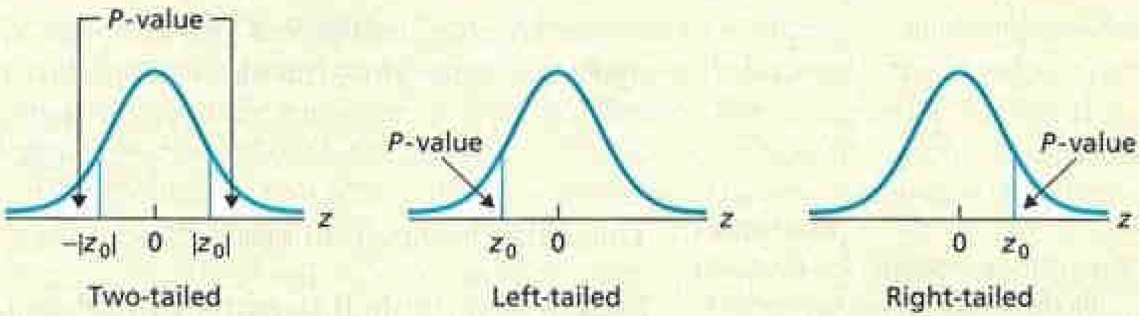
Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

and denote that value z_0.

Step 4 Use Table II to obtain the P-value.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Procedure 9.3A

The One-Sample t-Test for a Population Mean (Critical-Value Approach)

Assumptions

1. Normal population or large sample
2. σ unknown

Step 1 **The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is**

H_a: $\mu \neq \mu_0$ (Two-tailed)	or	H_a: $\mu < \mu_0$ (Left-tailed)	or	H_a: $\mu > \mu_0$ (Right-tailed)

Step 2 **Decide on the significance level, α.**

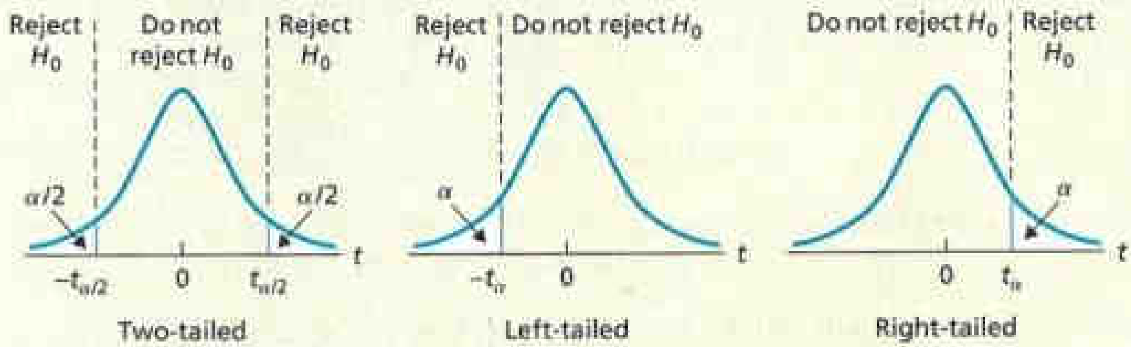
Step 3 **Compute the value of the test statistic**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}.$$

Step 4 **The critical value(s) are**

$\pm t_{\alpha/2}$ (Two-tailed)	or	$-t_{\alpha}$ (Left-tailed)	or	t_{α} (Right-tailed)

with df $= n - 1$. Use Table IV to find the critical value(s).



Step 5 **If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.**

Step 6 **Interpret the results of the hypothesis test.**

The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Procedure 9.3B

The One-Sample t-Test for a Population Mean (P-Value Approach)

Assumptions

1. Normal population or large sample
2. σ unknown

Step 1 The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is

H_a: $\mu \neq \mu_0$ (Two-tailed)	or	H_a: $\mu < \mu_0$ (Left-tailed)	or	H_a: $\mu > \mu_0$ (Right-tailed)

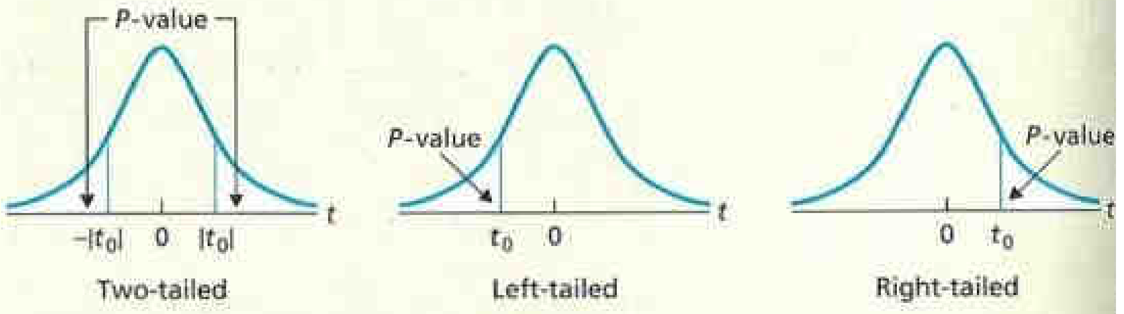
Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

and denote that value t_0.

Step 4 The t-statistic has df $= n - 1$. Use Table IV to estimate the P-value, or obtain it exactly by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Solution

TABLE 9.11
pH levels for 15 lakes

7.2	7.3	6.1	6.9	6.6
7.3	6.3	5.5	6.3	6.5
5.7	6.9	6.7	7.9	5.8

Step 1 State the null and alternative hypotheses.

Let μ denote the mean pH level of all high mountain lakes in the Southern Alps. Then the null and alternative hypotheses are

$$H_0: \mu = 6 \text{ (mean pH level is not greater than 6)}$$

$$H_a: \mu > 6 \text{ (mean pH level is greater than 6).}$$

Note that the hypothesis test is right-tailed because a greater-than sign (>) appears in the alternative hypothesis.

Step 2 Decide on the significance level, α.

We are to perform the test at the 5% significance level, so $\alpha = 0.05$.

Step 3 **Compute the value of the test statistic**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}.$$

We have $\mu_0 = 6$ and $n = 15$ and calculate the mean and standard deviation of the sample data in Table 9.11 as 6.6 and 0.672, respectively. Hence the value of the test statistic is

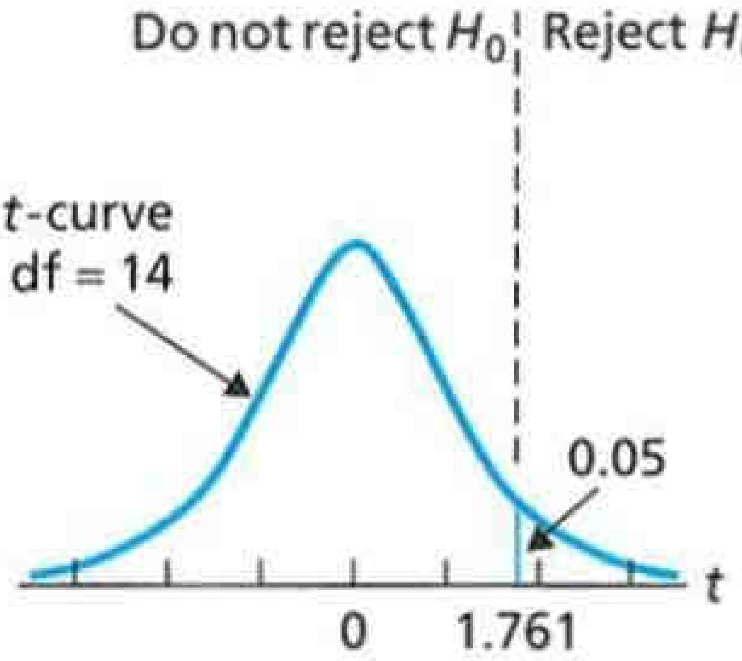
$$t = \frac{6.6 - 6}{0.672/\sqrt{15}} = 3.458.$$

Critical-Value Approach

Step 4 **The critical value for a right-tailed test is t_α, with df $= n - 1$.**

We have $n = 15$ and $\alpha = 0.05$. Table IV shows that for df $= 15 - 1 = 14$, $t_{0.05} = 1.761$, as shown in Fig. 9.21A.

FIGURE 9.21A



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

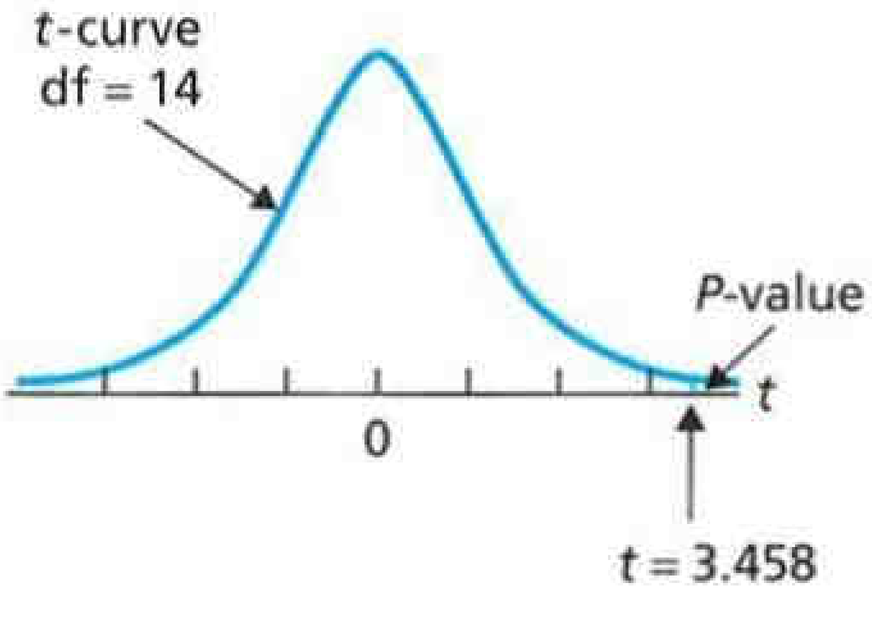
The value of the test statistic, found in Step 3, is $t = 3.458$. Figure 9.21A reveals that it falls in the rejection region. Consequently, we reject H_0. The test results are statistically significant at the 5% level.

P-Value Approach

Step 4 The t-statistic has df $= n - 1$. Use Table IV to estimate the *P*-value, or obtain it exactly by using technology.

From Step 3, the value of the test statistic is $t = 3.458$. The test is right-tailed, so the *P*-value is the probability of observing a value of t of 3.458 or greater if the null hypothesis is true. That probability equals the shaded area in Fig. 9.21B.

FIGURE 9.21B



We have $n = 15$, and so df $= 15 - 1 = 14$. From Fig. 9.21B and Table IV, $P < 0.005$. (Using technology, we obtain $P = 0.00192$.)

Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

From Step 4, $P < 0.005$. Because the P-value is less than the specified significance level of 0.05, we reject H_0. The test results are statistically significant at the 5% level and (see Table 9.10 on page 405) provide very strong evidence against the null hypothesis.

Step 6 **Interpret the results of the hypothesis test.**

What Does it Mean?

At the 5% significance level, the data provide sufficient evidence to conclude that, on average, high mountain lakes in the Southern Alps are nonacidic.