Linear Programming Examples

1. Selecting among alternative projects

Choosing among 7 alternative projects, based on the available budget and their Net Present Value (NPV)

Year	Projects							Budget
	1	2	3	4	5	6	7	
1	40	20	25	80	20	90	50	250
2	10	30	30	40	20	25	10	125
3	25	0	20	30	20	0	0	75
4	25	0	10	0	10	10	30	50
5	10	35	0	15	10	20	0	50
NPV	250	180	225	300	150	275	200	

Decision variables

x_i = 1 if project i is selected or 0 if project i is not selected

Objective function ~ Maximize total (for all 7 projects) NPV

$$\max \quad z = 250\,x_1 + 180\,x_2 + 225\,x_3 + 300\,x_4 + 150\,x_5 + 275\,x_6 + 200\,x_7$$

Constraints ~ Do not exceed available budget for every one of the 5 years

$$\begin{aligned}
\text{subject to} \quad & 40\,x_1 + 20\,x_2 + 25\,x_3 + 80\,x_4 + 20\,x_5 + 90\,x_6 + 50\,x_7 \leqq 250 && \text{Year 1} \\
& 10\,x_1 + 30\,x_2 + 30\,x_3 + 40\,x_4 + 20\,x_5 + 25\,x_6 + 10\,x_7 \leqq 125 && \text{Year 2} \\
& 25\,x_1 + 20\,x_3 + 30\,x_4 + 20\,x_5 \leqq 75 && \text{Year 3} \\
& 25\,x_1 + 10\,x_3 + 10\,x_5 + 10\,x_6 + 30\,x_7 \leqq 50 && \text{Year 4} \\
& 10\,x_1 + 35\,x_2 + 15\,x_4 + 10\,x_5 + 20\,x_6 \leqq 50 && \text{Year 5}
\end{aligned}$$

x_i binary (0 or 1) for i=1 to 7

Integer (binary) LP

Microsoft Excel Solver solution

	P1	P2	P3	P4	P5	P6	P7	Budget
Y1	40	20	25	80	20	90	50	250
Y2	10	30	30	40	20	25	10	125
Y3	25	0	20	30	20	0	0	75
Y4	25	0	10	0	10	10	30	50
Y5	10	35	0	15	10	20	0	50
NPV	250	180	225	300	150	275	200	
	x1	x2	x3	x4	x5	x6	x7	
	1	0	1	1	0	1	0	
max z=	1050							
s.t.	235	<=	250	Y1				
	105	<=	125	Y2				
	75	<=	75	Y3				
	45	<=	50	Y4				
	45	<=	50	Y5				

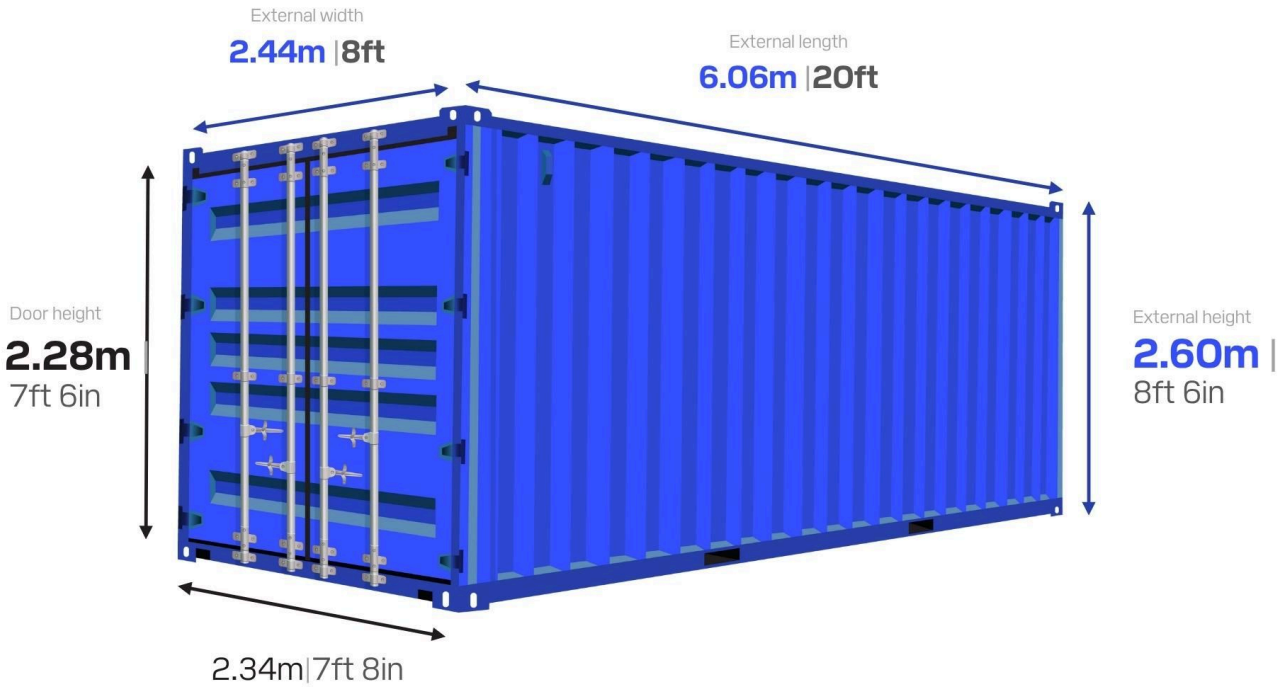
2. Scheduling alternative vessel types

A shipping company operates two types of vessels on a regional route:

	Vessel A	Vessel B
Cargo capacity	120 TEU	300 TEU
Operating cost per trip	$18 000	$40 000
Net profit per trip	$32 000	$70 000

TEU = Twenty-Foot Equivalent Unit

- Standard unit of measure for containerized cargo capacity in maritime industry
- One TEU = volume of a standard 20-foot shipping container
    - A 20-foot container = 1 TEU
    - A 40-foot container = 2 TEU



20ft	External length	External height	External width	Internal length	Internal height	Internal width	Door width	Door height	Volume	Max. gross weight	Tare weight
	6.06m	2.60m	2.44m	5.9m	2.39m	2.35m	2.34m	2.28m	33.2m3	30,480kg	2,000kg
	20ft	8ft 6in	8ft	19ft 4in	7ft 10in	7ft 9in	7ft 8in	7ft 6in	1,172cf	67,196lbs	4,409lbs

Demand forecast: transport at least 900 TEU/week

Budgetary constraint: total operating costs must not exceed $300,000/week

What is the optimal number of trips/week for each vessel?

- Fleet deployment
- Capacity planning

- Liner shipping scheduling
- Short-sea route optimization

Decision variables

x_1 = number of trips/week by Vessel A
x_2 = number of trips/week by Vessel B

Objective function ~ Maximize total weekly profit

$$\text{Max } z = 32\,000\,x_1 + 70\,000\,x_2$$

Constraints

subject to	$120\,x_1 + 300\,x_2 \geq 900$	Cargo capacity
	$18\,000\,x_1 + 40\,000\,x_2 \leq 300\,000$	Operating cost
	$x_1 \geq 0, x_2 \geq 0$	Non-negativity
	x_1, x_2 integer	

Integer LP (ILP)

Microsoft Excel Solver solution

				TEU	Operating cost	Net profit
	x1	10		120	18000	32000
	x2	3		300	40000	70000
	max z =	530000				
s.t.	2100	>=	900			
	300000	<=	300000			