

This problem is another example (like the Wyndor problem) of a *resource-allocation problem*. The first three categories of constraints all are *resource constraints*. The fourth category then adds some side constraints.

Controlling Air Pollution

The NORI & LEETS CO., one of the major producers of steel in its part of the world, is located in the city of Steeltown and is the only large employer there. Steeltown has grown and prospered along with the company, which now employs nearly 50,000 residents. Therefore, the attitude of the townspeople always has been, What's good for Nori & Leets is good for the town. However, this attitude is now changing; uncontrolled air pollution from the company's furnaces is ruining the appearance of the city and endangering the health of its residents.

A recent stockholders' revolt resulted in the election of a new enlightened board of directors for the company. These directors are determined to follow socially responsible policies, and they have been discussing with Steeltown city officials and citizens' groups what to do about the air pollution problem. Together they have worked out stringent air quality standards for the Steeltown airshed.

The three main types of pollutants in this airshed are particulate matter, sulfur oxides, and hydrocarbons. The new standards require that the company reduce its annual emission of these pollutants by the amounts shown in Table 3.12. The board of directors has instructed management to have the engineering staff determine how to achieve these reductions in the most economical way.

The steelworks has two primary sources of pollution, namely, the blast furnaces for making pig iron and the open-hearth furnaces for changing iron into steel. In both cases the engineers have decided that the most effective types of abatement methods are (1) increasing the height of the smokestacks,⁶ (2) using filter devices (including gas traps) in the smokestacks, and (3) including cleaner, high-grade materials among the fuels for the furnaces. Each of these methods has a technological limit on how heavily it can be used (e.g., a maximum feasible increase in the height of the smokestacks), but there also is considerable flexibility for using the method at a fraction of its technological limit.

Table 3.13 shows how much emission (in millions of pounds per year) can be eliminated from each type of furnace by fully using any abatement method to its technological limit. For purposes of analysis, it is assumed that each method also can be used less fully to achieve any fraction of the emission-rate reductions shown in this table. Furthermore, the fractions can be different for blast furnaces and for open-hearth furnaces. For either type of furnace, the emission reduction achieved by each method is not substantially affected by whether the other methods also are used.

After these data were developed, it became clear that no single method by itself could achieve all the required reductions. On the other hand, combining all three methods at full capacity on both types of furnaces (which would be prohibitively expensive if the company's

■ **TABLE 3.12** Clean air standards for the Nori & Leets Co.

Pollutant	Required Reduction in Annual Emission Rate (Million Pounds)
Particulates	60
Sulfur oxides	150
Hydrocarbons	125

⁶This particular abatement method has become a controversial one. Because its effect is to reduce ground-level pollution by spreading emissions over a greater distance, environmental groups contend that this creates more acid rain by keeping sulfur oxides in the air longer. Consequently, the U.S. Environmental Protection Agency adopted new rules in 1985 to remove incentives for using tall smokestacks.

■ **TABLE 3.13** Reduction in emission rate (in millions of pounds per year) from the maximum feasible use of an abatement method for Nori & Leeds Co.

Pollutant	Taller Smokestacks		Filters		Better Fuels	
	Blast Furnaces	Open-Hearth Furnaces	Blast Furnaces	Open-Hearth Furnaces	Blast Furnaces	Open-Hearth Furnaces
Particulates	12	9	25	20	17	13
Sulfur oxides	35	42	18	31	56	49
Hydrocarbons	37	53	28	24	29	20

products are to remain competitively priced) is much more than adequate. Therefore, the engineers concluded that they would have to use some combination of the methods, perhaps with fractional capacities, based upon the relative costs. Furthermore, because of the differences between the blast and the open-hearth furnaces, the two types probably should not use the same combination.

An analysis was conducted to estimate the total annual cost that would be incurred by each abatement method. A method's annual cost includes increased operating and maintenance expenses as well as reduced revenue due to any loss in the efficiency of the production process caused by using the method. The other major cost is the *start-up cost* (the initial capital outlay) required to install the method. To make this one-time cost commensurable with the ongoing annual costs, the time value of money was used to calculate the annual expenditure (over the expected life of the method) that would be equivalent in value to this start-up cost.

This analysis led to the total annual cost estimates (in millions of dollars) given in Table 3.14 for using the methods at their full abatement capacities. It also was determined that the cost of a method being used at a lower level is roughly proportional to the fraction of the abatement capacity given in Table 3.13 that is achieved. Thus, for any given fraction achieved, the total annual cost would be roughly that fraction of the corresponding quantity in Table 3.14.

The stage now was set to develop the general framework of the company's plan for pollution abatement. This plan specifies which types of abatement methods will be used and at what fractions of their abatement capacities for (1) the blast furnaces and (2) the open-hearth furnaces. Because of the combinatorial nature of the problem of finding a plan that satisfies the requirements with the smallest possible cost, an OR team was formed to solve the problem. The team adopted a linear programming approach, formulating the model summarized next.

Formulation as a Linear Programming Problem. This problem has six decision variables x_j , $j = 1, 2, \dots, 6$, each representing the use of one of the three abatement methods for one of the two types of furnaces, expressed as a *fraction of the abatement capacity* (so x_j cannot exceed 1). The ordering of these variables is shown in Table 3.15. Because the

■ **TABLE 3.14** Total annual cost from the maximum feasible use of an abatement method for Nori & Leeds Co. (\$ millions)

Abatement Method	Blast Furnaces	Open-Hearth Furnaces
Taller smokestacks	8	10
Filters	7	6
Better fuels	11	9

■ **TABLE 3.15** Decision variables (fraction of the maximum feasible use of an abatement method) for Nori & Leets Co.

Abatement Method	Blast Furnaces	Open-Hearth Furnaces
Taller smokestacks	x_1	x_2
Filters	x_3	x_4
Better fuels	x_5	x_6

objective is to minimize total cost while satisfying the emission reduction requirements, the data in Tables 3.12, 3.13, and 3.14 yield the following model:

$$\text{Minimize } Z = 8x_1 + 10x_2 + 7x_3 + 6x_4 + 11x_5 + 9x_6,$$

subject to the following constraints:

1. Emission reduction:

$$12x_1 + 9x_2 + 25x_3 + 20x_4 + 17x_5 + 13x_6 \geq 60$$

$$35x_1 + 42x_2 + 18x_3 + 31x_4 + 56x_5 + 49x_6 \geq 150$$

$$37x_1 + 53x_2 + 28x_3 + 24x_4 + 29x_5 + 20x_6 \geq 125$$

2. Technological limit:

$$x_j \leq 1, \quad \text{for } j = 1, 2, \dots, 6$$

3. Nonnegativity:

$$x_j \geq 0, \quad \text{for } j = 1, 2, \dots, 6.$$

The OR team used this model⁷ to find a minimum-cost plan

$$(x_1, x_2, x_3, x_4, x_5, x_6) = (1, 0.623, 0.343, 1, 0.048, 1),$$

with $Z = 32.16$ (total annual cost of \$32.16 million). Sensitivity analysis then was conducted to explore the effect of making possible adjustments in the air standards given in Table 3.12, as well as to check on the effect of any inaccuracies in the cost data given in Table 3.14. (This story is continued in Case 7.1 at the end of Chap. 7.) Next came detailed planning and managerial review. Soon after, this program for controlling air pollution was fully implemented by the company, and the citizens of Steeltown breathed deep (cleaner) sighs of relief.

Like the radiation therapy problem, this is another example of a *cost–benefit–trade-off problem*. The cost in this case is a monetary cost and the benefits are the various types of pollution abatement. The benefit constraint for each type of pollutant has the amount of abatement achieved on the left-hand side and the minimum acceptable level of abatement on the right-hand side.

Reclaiming Solid Wastes

The SAVE-IT COMPANY operates a reclamation center that collects four types of solid waste materials and treats them so that they can be amalgamated into a salable product. (Treating and amalgamating are separate processes.) Three different grades of this product can be made (see the first column of Table 3.16), depending upon the mix of the materials used. Although there is some flexibility in the mix for each grade, quality standards may specify the minimum or maximum amount allowed for the proportion of a material in the

⁷An equivalent formulation can express each decision variable in natural units for its abatement method; for example, x_1 and x_2 could represent the number of *feet* that the heights of the smokestacks are increased.