AREA UNDER THE NORMAL CURVE

Subroutine: NORMDT

Description

Computes the area under the upper and lower tails of the standard normal curve.

Method

The integrals

$$N(z) = (1/2\pi)^{\frac{1}{2}} \int_{z}^{\infty} \exp(-x^2/2)\mathrm{d}x \quad \text{and}$$

$$N(z) = (1/2\pi)^{\frac{1}{2}} \int_{-\infty}^{z} \exp(-x^2/2)\mathrm{d}x$$

are evaluated using two approximations:

$$N(z) = 0.5 - zf(z^2) \quad \text{when } |z| \leqslant 1.28 \text{ and}$$
$$N(z) = (1/2\pi)^{\frac{1}{2}} \exp(-z^2/2)f(z) \quad \text{when } |z| > 1.28$$

This technique is based on the work published by Adams (Adams, 1969) and subsequently adapted and published in *Applied Statistics Algorithms* (Griffiths and Hill, 1985a).

Subroutine Listing

```
2000 REM      AREA UNDER THE NORMAL CURVE
2010 REM
2020 REM      SUBROUTINE: NORMDT
2030 REM
2040 REM      COMPUTES BOTH UPPER AND LOWER TAIL AREAS
2050 REM      UNDER THE STANDARD NORMAL N(0,1) CURVE
2060 REM
2070 REM      VARIABLES:
2080 REM       INPUT:
2090 REM        Z.......VALUE OF NORMAL VARIATE
2100 REM       OUTPUT:
2110 REM        LAREA...AREA FROM MINUS INFINITY TO Z
2120 REM        UAREA...AREA FROM Z TO PLUS INFINITY
2130 REM       LOCAL:...AZ, FAZ, NORMA, V1
2140 REM
2150 REM      SET INITIAL CONDITIONS AND TEST
2160 REM      FOR LARGE ABS(Z)
2170    LET AZ = ABS(Z)
2180    LET NORMA = 0
```

```
2190     IF AZ > 12 GOTO 2400
2200      LET FAZ = 0.5*AZ*AZ
2210      IF AZ <= 1.28 THEN GOTO 2330
2220 REM
2230 REM       COMPUTE FOR ABS(Z) > 1.28
2240     LET FAZ = 0.398942280385*EXP(-FAZ)
2250     LET V1 = AZ + 3.99019417011
2260     LET V1 = AZ + 0.742380924027 + 30.789933034/V1
2270     LET V1 = AZ + 4.8385912808 - 15.1508972451/V1
2280     LET V1 = AZ - 0.151679116635 + 5.29330324926/V1
2290     LET V1 = AZ + 3.98064794E-4 + 1.98615381364/V1
2300     LET V1 = 1.00000615302/V1
2310     LET NORMA = FAZ/(AZ - 3.8052E-8 + V1)
2320     GOTO 2400
2330 REM
2340 REM       COMPUTE FOR ABS(Z) <= 1.28
2350     LET V1 = FAZ + 5.92885724438
2360     LET V1 = FAZ + 2.62433121679 + 48.6959930692/V1
2370     LET V1 = FAZ + 5.75885480458 - 29.8213557808/V1
2380     LET V1 = 0.399903438504*FAZ/V1
2390     LET NORMA = 0.5 - AZ*(0.398942280444 - V1)
2400 REM
2410 REM       SET LAREA AND UAREA FOR -VE AND +VE Z
2420     IF Z >= 0 THEN GOTO 2460
2430      LAREA = NORMA
2440      UAREA = 1 - NORMA
2450      GOTO  2490
2460 REM       Z >= 0
2470      LAREA = 1 - NORMA
2480      UAREA = NORMA
2490 REM
2500 RETURN
```

Sample Program

Determine the area under the lower and upper tails of the standard normal distribution for values of z in the range -6 to $+6$ at intervals of 0.5

```
1000 REM       SAMPLE PROGRAM: PNORMDT
1010 REM
1020 REM       TABULATES AREA UNDER THE NORMAL N(0,1) CURVE
1030 REM
1040 REM       VARIABLES:
1050 REM        N1.......MAXIMUM VALUE OF Z
1060 REM        N2.......VALUE OF STEP
1070 REM        N3.......LOWEST VALUE OF Z
1080 REM
1090 REM       SET INITIAL VALUES AND PRINT HEADINGS
1100     LET N1 = 6
```

```
1110    LET N2 = 0.5
1120    LET N3 = -6
1130    PRINT "      VARIATE", "      LOWER", "      UPPER"
1140    PRINT "            Z", "       AREA", "       AREA"
1150 REM
1160 REM      COMPUTE AREA UNDER NORMAL CURVE
1170 REM      CALL SUBROUTINE NORMDT
1180    FOR Z = N3 TO N1 STEP N2
1190       GOSUB 2000
1200       PRINT Z, LAREA, UAREA
1210    NEXT Z
1220 END
```

```
>RUN
     VARIATE       LOWER          UPPER
        Z          AREA           AREA
       -6 9.86588E-10                1
     -5.5  1.89896E-8                1
       -5  2.86652E-7                1
     -4.5  3.39767E-6         0.999997
       -4  3.16712E-5         0.999968
     -3.5  2.32629E-4         0.999767
       -3   1.3499E-3          0.99865
     -2.5  6.20967E-3          0.99379
       -2  2.27501E-2          0.97725
     -1.5  6.68072E-2         0.933193
       -1    0.158655         0.841345
     -0.5    0.308538         0.691462
        0         0.5              0.5
      0.5    0.691462         0.308538
        1    0.841345         0.158655
      1.5    0.933193       6.68072E-2
        2     0.97725       2.27501E-2
      2.5     0.99379       6.20967E-3
        3     0.99865        1.3499E-3
      3.5    0.999767       2.32629E-4
        4    0.999968       3.16712E-5
      4.5    0.999997       3.39767E-6
        5           1       2.86652E-7
      5.5           1       1.89896E-8
        6           1      9.86588E-10
```

The results agree with those published in five-figure tables – much higher accuracies are claimed for this method.

Notes

1. The test AZ > 12 may be adjusted to take account of precision of the working and the magnitude of the smallest number the machine can hold. This facility

will prove useful if unusually large values of z are likely to occur by design or in error and interruption of the program in run time proves undesirable.

2. For full accuracy it is important that the tail area selected for study is computed directly; that is, if $z > 0$, use UAREA to compute the small area and not $1 -$ LAREA. Similarly, if $z < 0$ use LAREA in preference to $1 -$ UAREA.