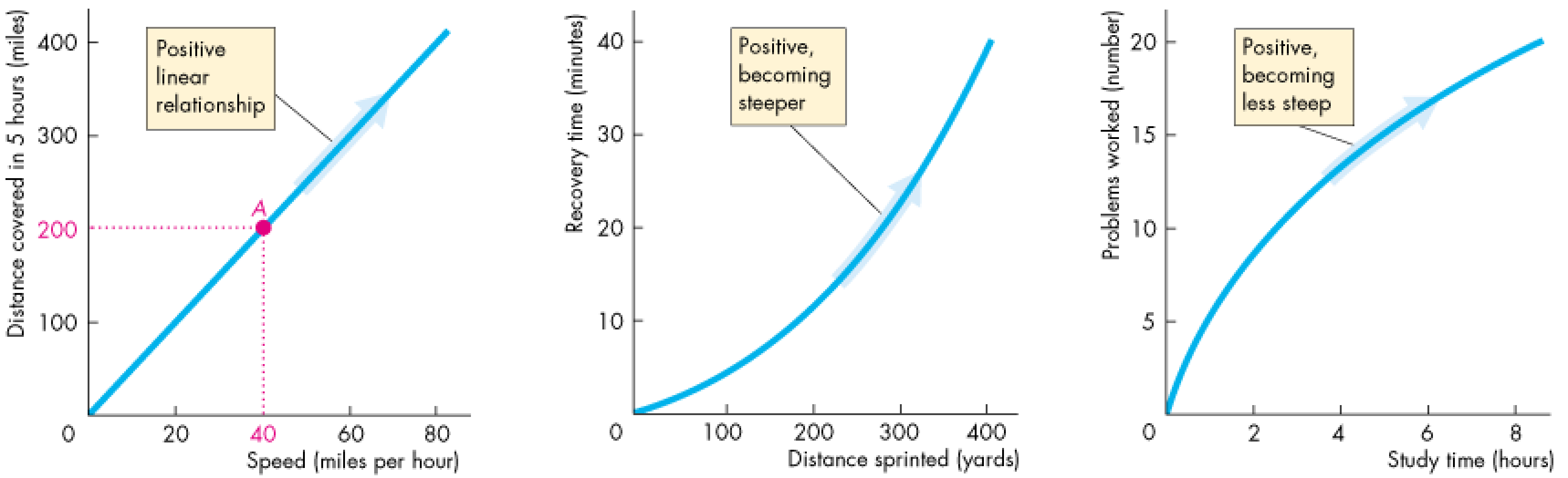
Notes on Maths and Economics for students facing background difficulties

Some References on Graphs in Economics

- https://community.plu.edu/~315j06/doc/graphing-economic.ppt
- http://studyingeconomics.ac.uk/tips-for-working-efficiently/maths-help/linear-and-simultaneous-equations/
- http://www.austincc.edu/powens/+BusEco/Lectures/lectures.htm#ch1
- http://mongmara.yolasite.com/resources/Math4BusinessandEconomics/Applied%20Mathematics%20for%20Business%20and%20Economics.pdf
- http://faculty.uaeu.ac.ae/jaelee/class/math115.html
- https://www.khanacademy.org/

Graphs Used in Economic Models

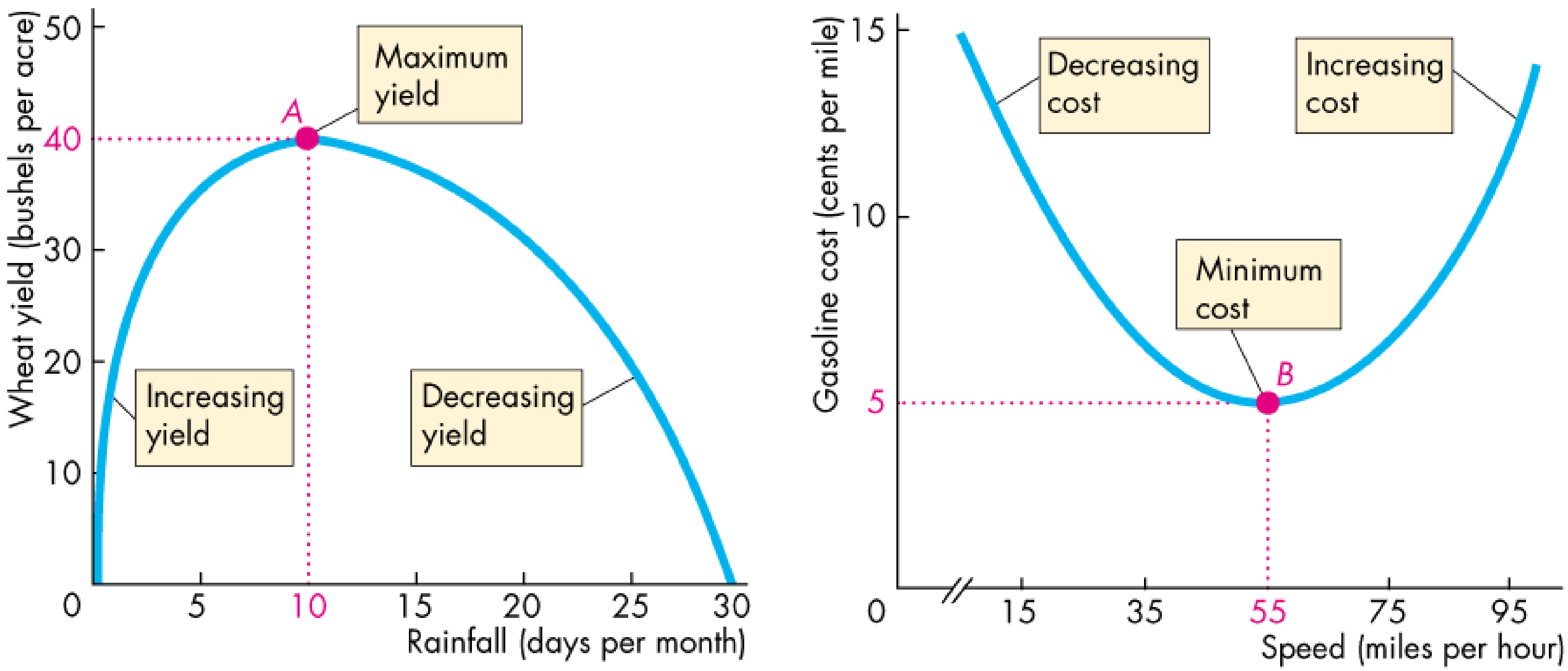


(a) Positive linear relationship **(b) Positive, becoming steeper** **(c) Positive, becoming less steep**

Graphs Used in Economic Models

Variables That Have a Maximum or a Minimum: The two graphs on the next slide show relationships that have a maximum and a minimum.

These relationships are positive over part of their range and negative over the other part.



(a) Relationship with a maximum

(b) Relationship with a minimum

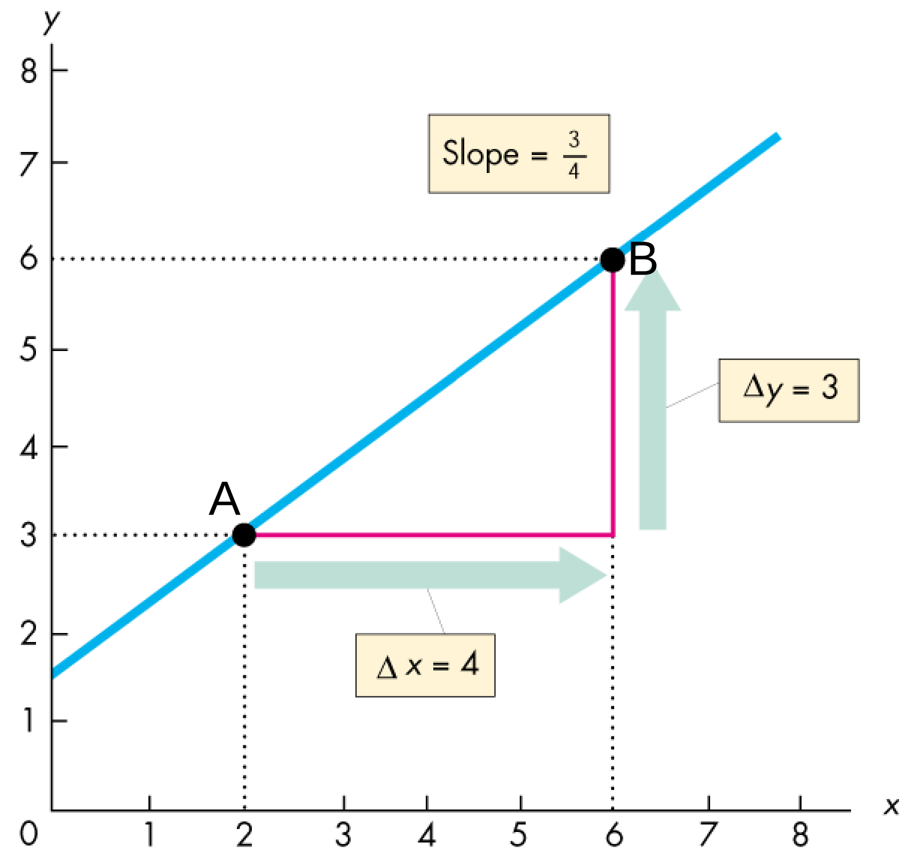
The Slope of a Relationship

The Slope of a Straight Line

The slope of a straight line is constant.

Graphically, the slope is calculated as the “rise” over the “run.”

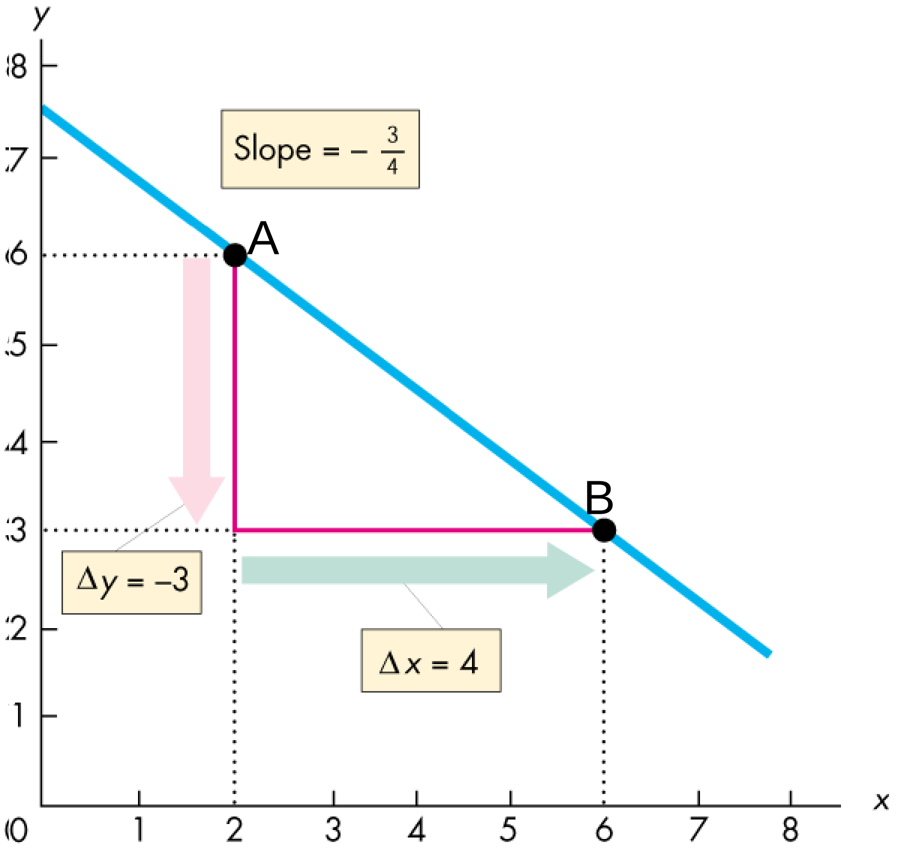
The slope is positive if the line is upward sloping.



(a) Positive slope

The Slope of a Relationship

The slope is negative if the line is downward sloping.



(b) Negative slope

The Slope of a Relationship

The Slope of a Curved Line

The slope of a curved line at a point varies depending on where along the curve it is calculated.

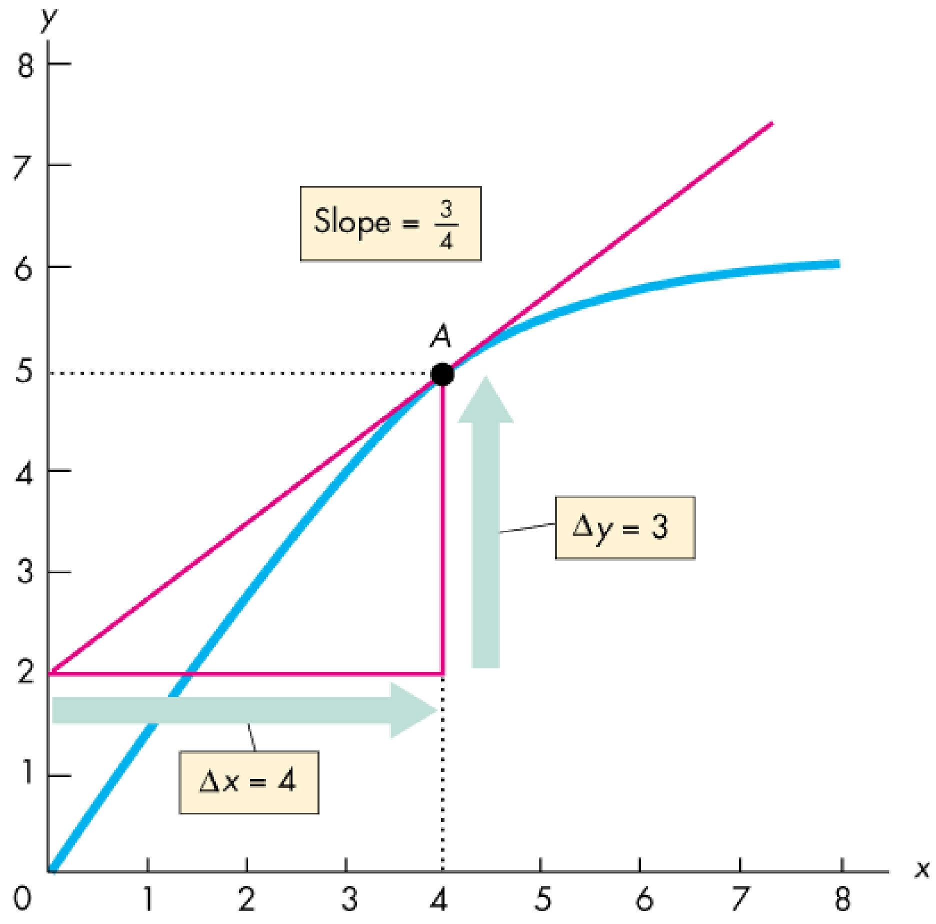
We can calculate the slope of a curved line either at a point or across an arc.

The Slope of a Relationship

Slope at a Point

The slope of a curved line at a point is equal to the slope of a straight line that is the tangent to that point.

Here, we calculate the slope of the curve at point *A*.

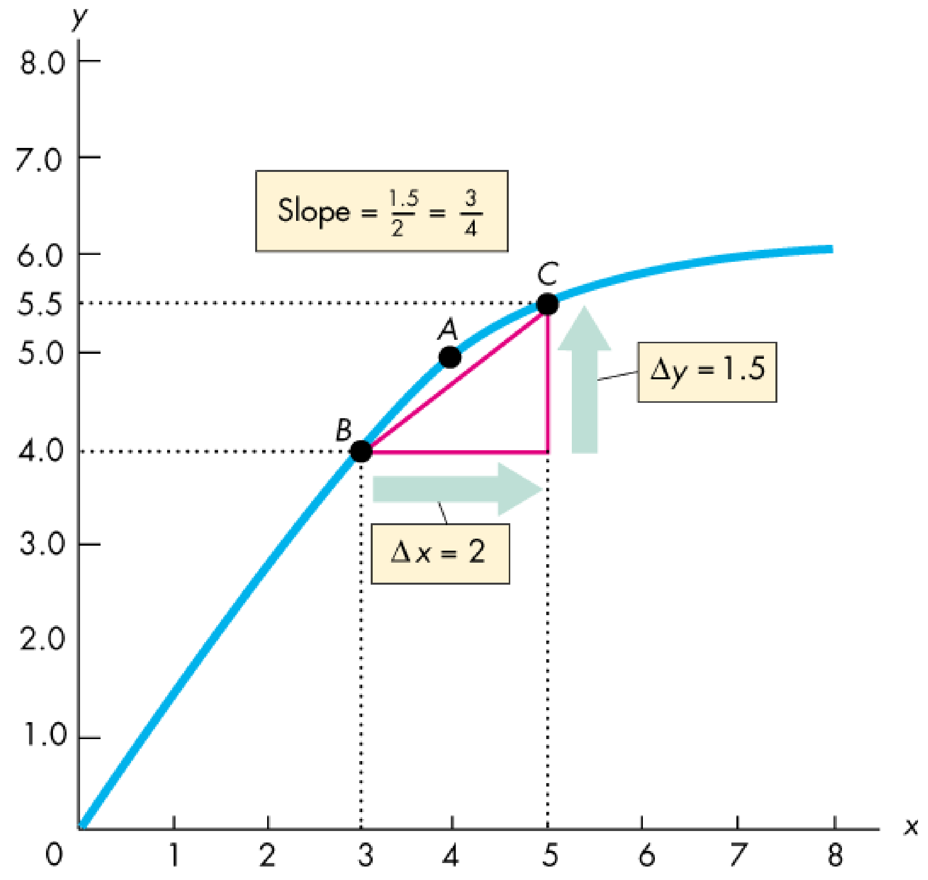


The Slope of a Relationship

Slope Across an Arc

The *average* slope of a curved line across an arc is equal to the slope of a straight line that joins the endpoints of the arc.

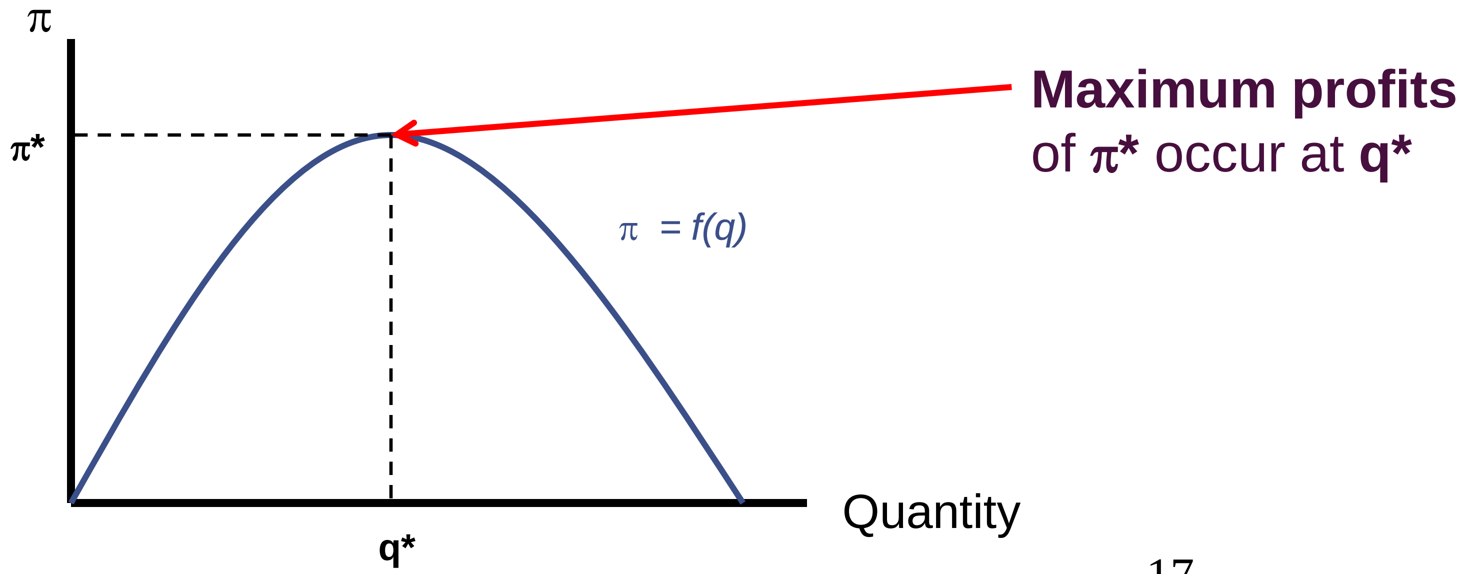
Here, we calculate the average slope of the curve along the arc *BC*.



Maximization of a Function of One Variable

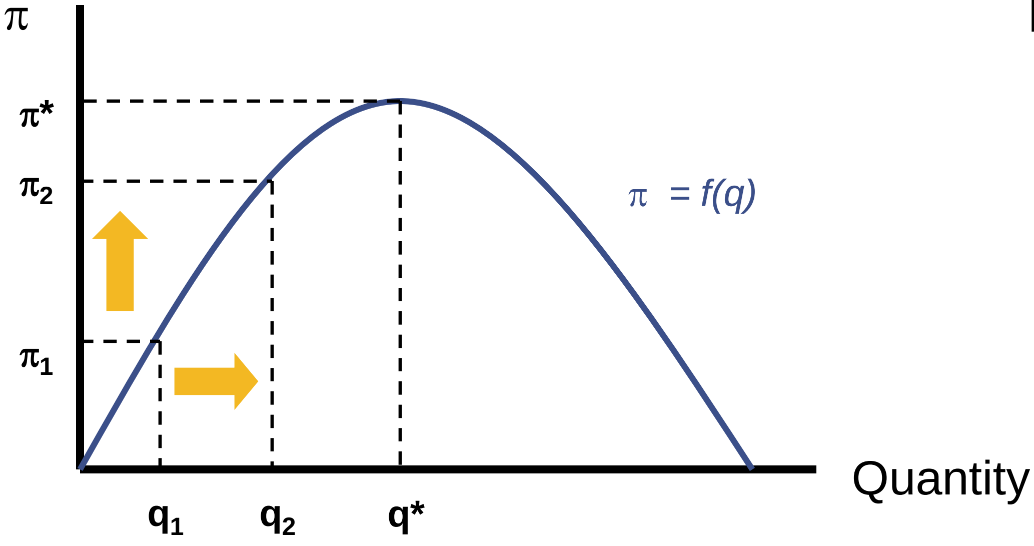
- Simple example: Manager of a firm wishes to **maximize profits**

$$\pi = f(q)$$



Maximization of a Function of One Variable

- The manager will likely try to vary q to see where the maximum profit occurs
  - an increase from q_1 to q_2 leads to a rise in π

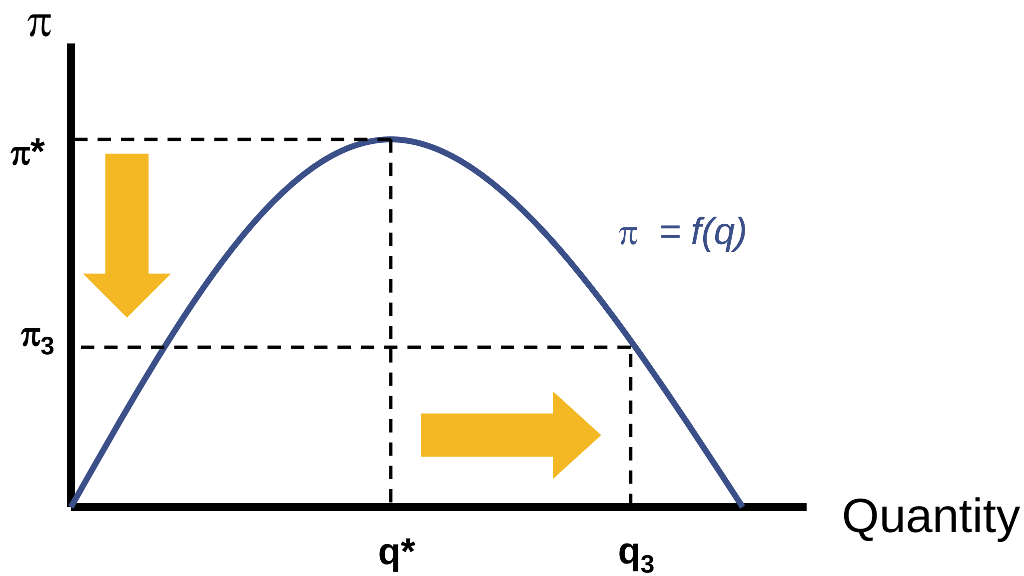


Derivative

$$\frac{\Delta\pi}{\Delta q} > 0$$

Maximization of a Function of One Variable

- If output is increased beyond q*, profit will decline
  - an increase from q^* to q_3 leads to a drop in π



$$\frac{\Delta \pi}{\Delta q} < 0$$

Value of a Derivative at a Point

- The evaluation of the derivative at the point $q = q_1$ can be denoted

$$\left.\frac{d\pi}{dq}\right|_{q=q_1}$$

- In our previous example,

Maximum

$$\left.\frac{d\pi}{dq}\right|_{q=q_1} > 0 \qquad \left.\frac{d\pi}{dq}\right|_{q=q_3} < 0 \qquad \left.\frac{d\pi}{dq}\right|_{q=q^*} = 0$$

First Order Condition for a Maximum

- For a function of one variable to attain its maximum value at some point, the derivative at that point must be zero

$$\left.\frac{df}{dq}\right|_{q=q^*} = 0$$

Rules for Finding Derivatives

1. If b is a constant, then $\frac{db}{dx} = 0$

2. If b is a constant, then $\frac{d[bf(x)]}{dx} = bf'(x)$

3. If b is constant, then $\frac{dx^b}{dx} = bx^{b-1}$

4. $\frac{d\ln x}{dx} = \frac{1}{x}$

Rules for Finding Derivatives

5. $\frac{da^x}{dx} = a^x \ln a$ for any constant a

– a special case of this rule is $de^x/dx = e^x$

Rules for Finding Derivatives

- Suppose that *f*(*x*) and *g*(*x*) are two functions of *x* and *f'*(*x*) and *g'*(*x*) exist
- Then

$$6.\ \frac{d[f(x) \pm g(x)]}{dx} = f'(x) \pm g'(x)$$

$$7.\ \frac{d[f(x) \cdot g(x)]}{dx} = f(x)g'(x) + f'(x)g(x)$$

Rules for Finding Derivatives

8. $$\frac{d\left(\frac{f(x)}{g(x)}\right)}{dx} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

provided that $g(x) \neq 0$

Example 1: of Profit Maximization (Rule 6)

- Suppose that the relationship between profit and output is

$$\pi = 1{,}000q - 5q^2$$

- The first order condition for a maximum is

$$d\pi/dq = 1{,}000 - 10q = 0$$

$$q^* = 100$$

Example 2: (Rule 6 of derivatives)

- **Let,**
- **Total Revenue TR**= $-0.75Q^2 + 200Q$,
- **Total Cost TC**= $500-10Q+2Q^2$
- **Total Profits π** =TR-TC, **Find Max Profits**.

Max Profits requires (π)'=0 i.e. first derivative to be equal to zero, which implies (TR-TC)'= 0 TR' - TC' = 0 **TR'=TC'** **(1)**

However, TR' =MR (Marginal Revenue), TC'= MC (Marginal Cost)

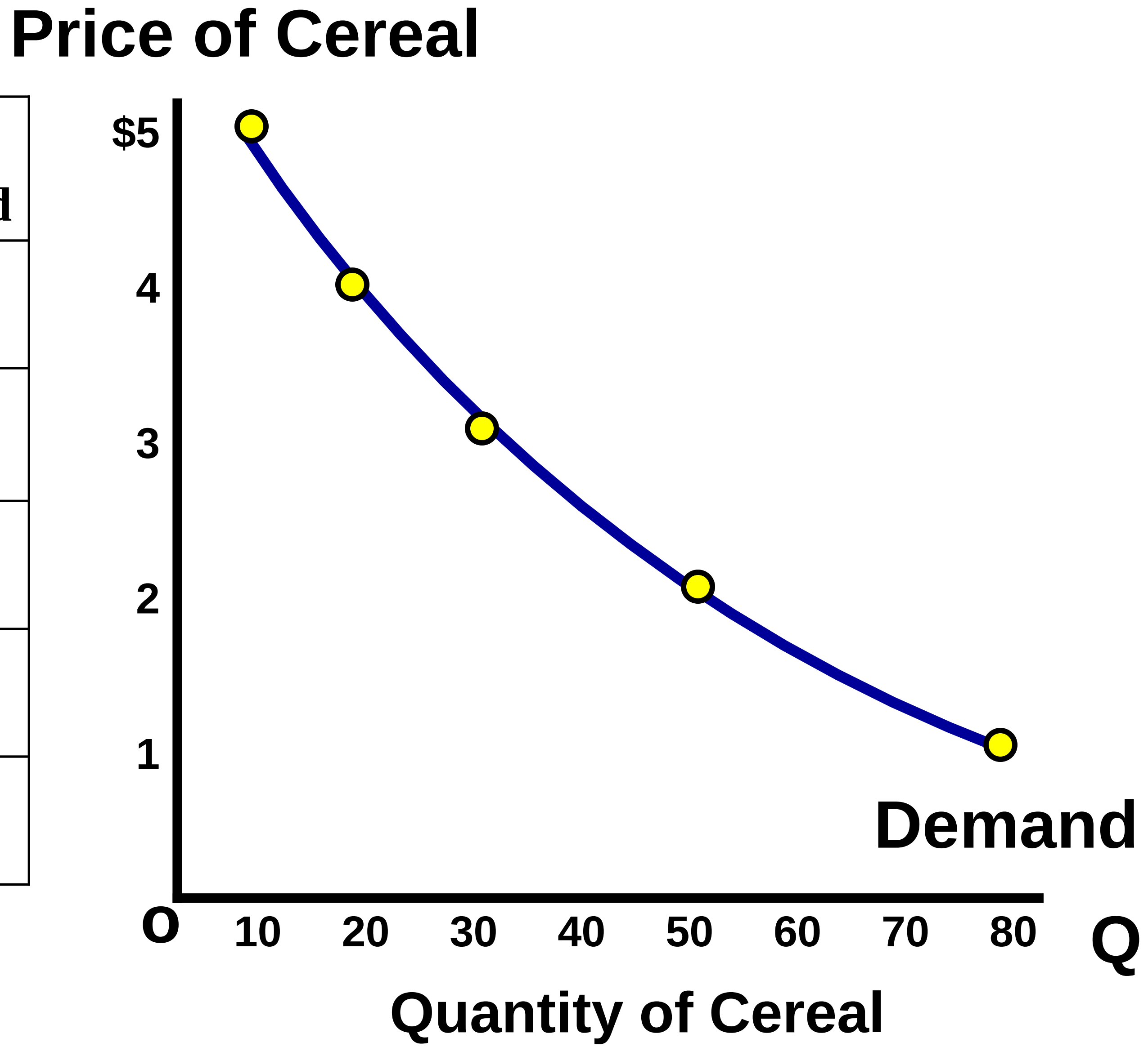
Therefore max Profits requires, **MR =MC** **(2)**

MR = $(-1.5Q + 200)$ and **MC=** $-10+4Q$

or $(-1.5Q + 200) = -10+4Q$ Solving for Q we get that **Q=38.2 is the quantity that maximises profits**

Demand Schedule

Price	Quantity Demanded
$5	10
$4	20
$3	30
$2	50
$1	80



How to get the Market Demand / add the demand of each consumer at the market equilibrium point of each consumer

Billy

Price	Q Demd
$5	1
$4	2
$3	3
$2	5
$1	7

+

Jean

Price	Q Demd
$5	0
$4	1
$3	2
$2	3
$1	5

+

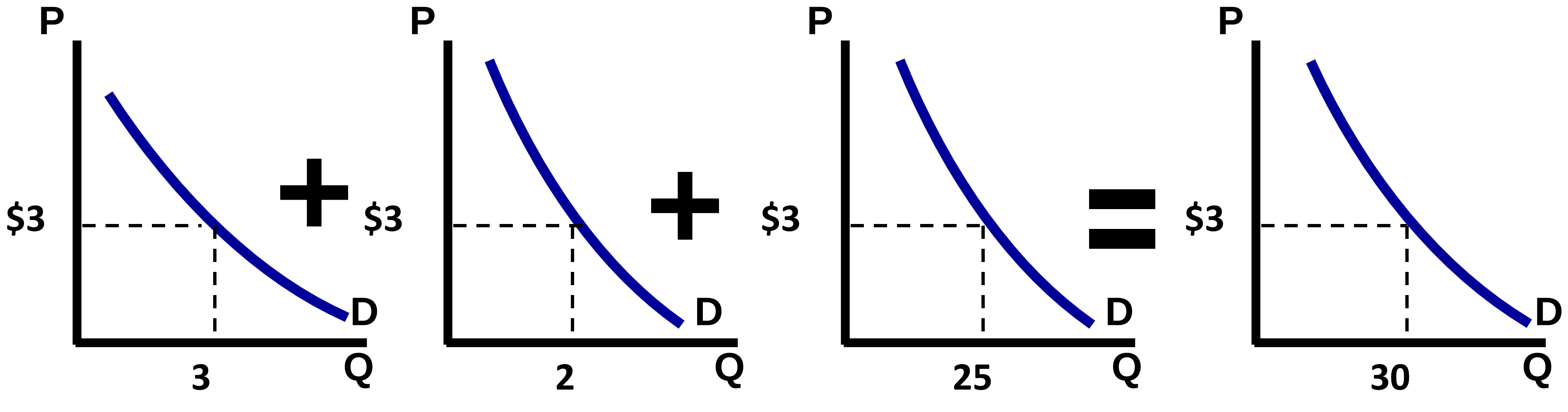
Other Individuals

Price	Q Demd
$5	9
$4	17
$3	25
$2	42
$1	68

=

Market

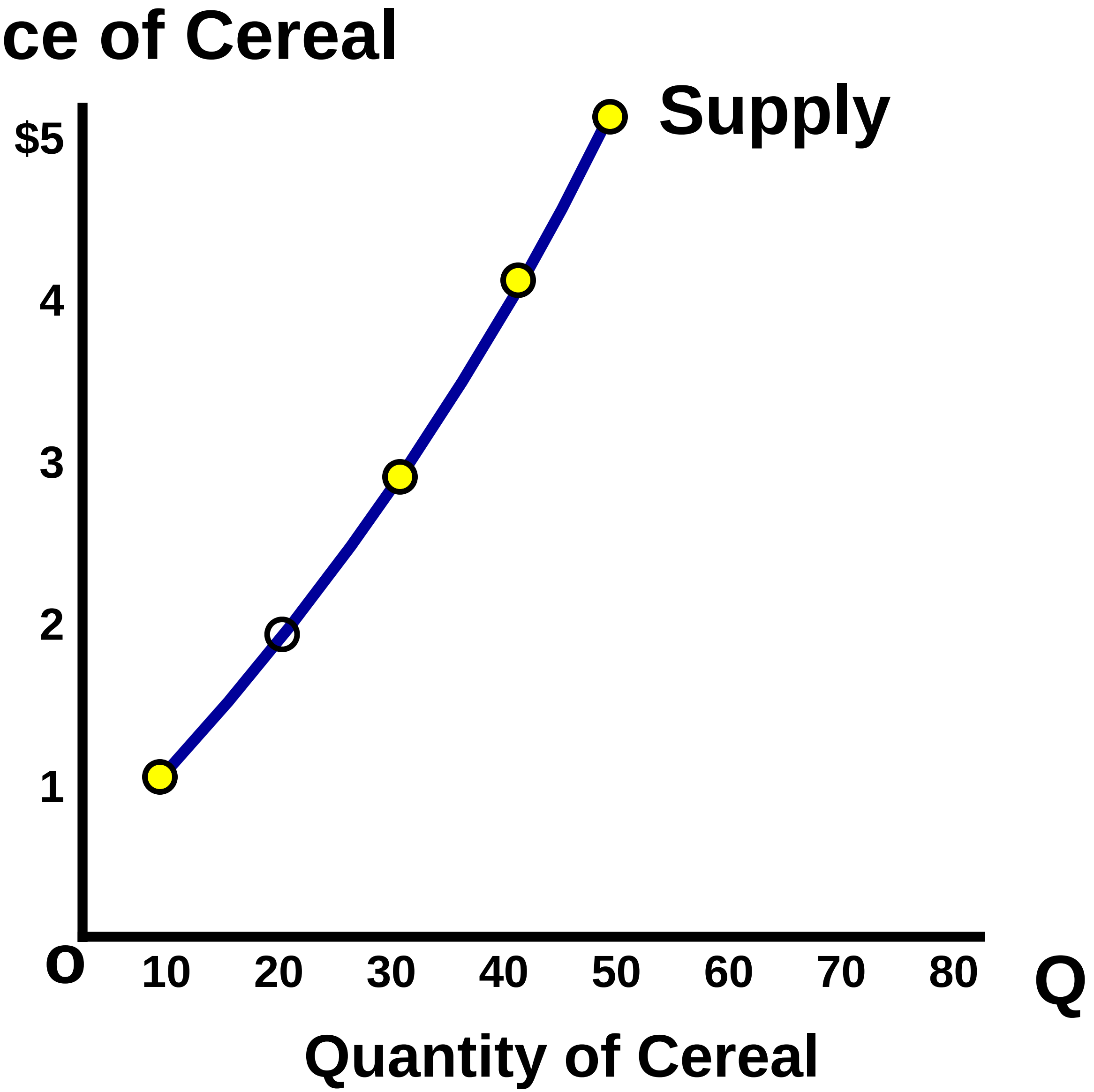
Price	Q Demd
$5	10
$4	20
$3	30
$2	50
$1	80



GRAPHING SUPPLY

Supply Schedule

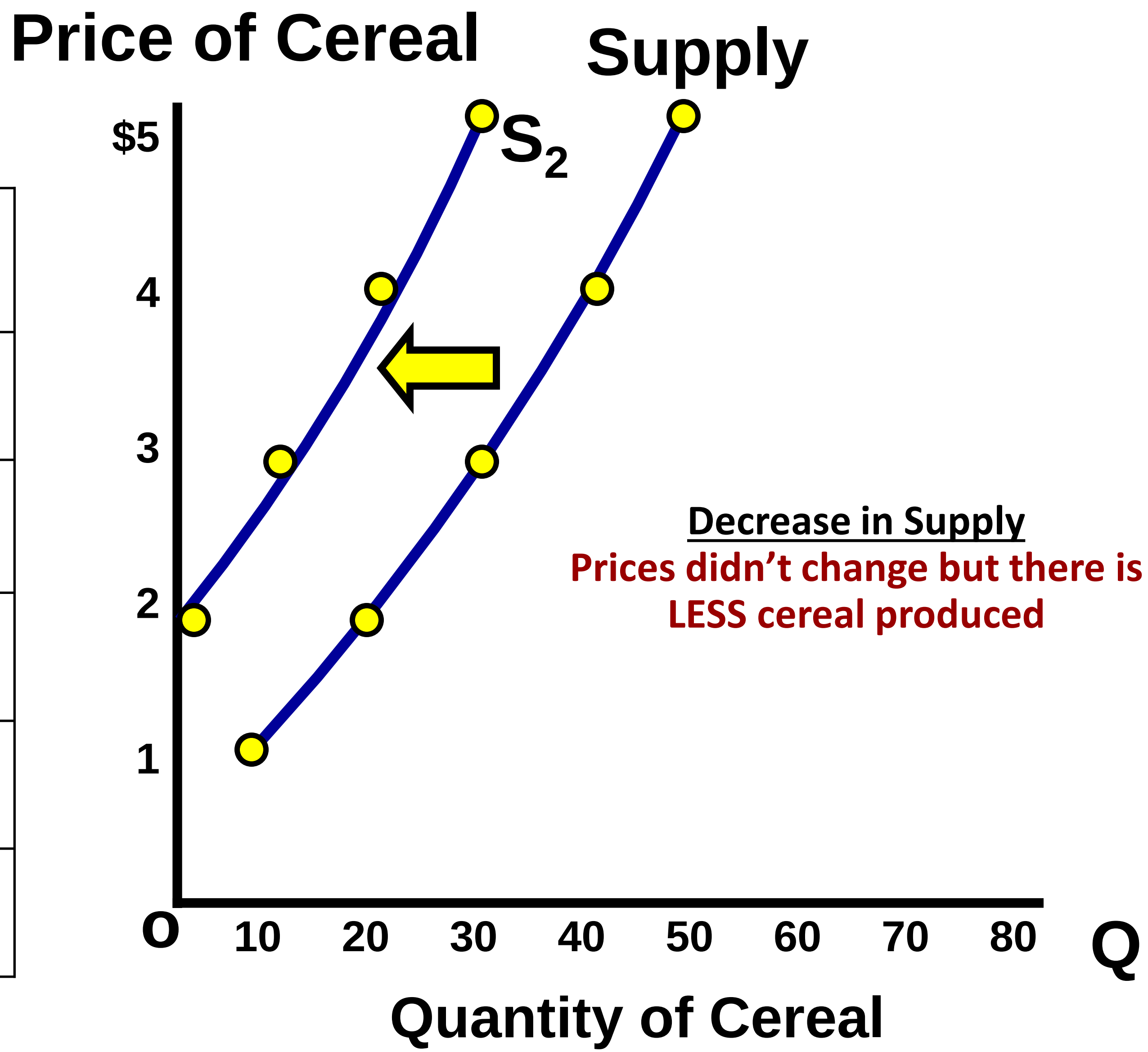
Price	Quantity Supplied
$5	50
$4	40
$3	30
$2	20
$1	10



Change in Supply

Supply Schedule

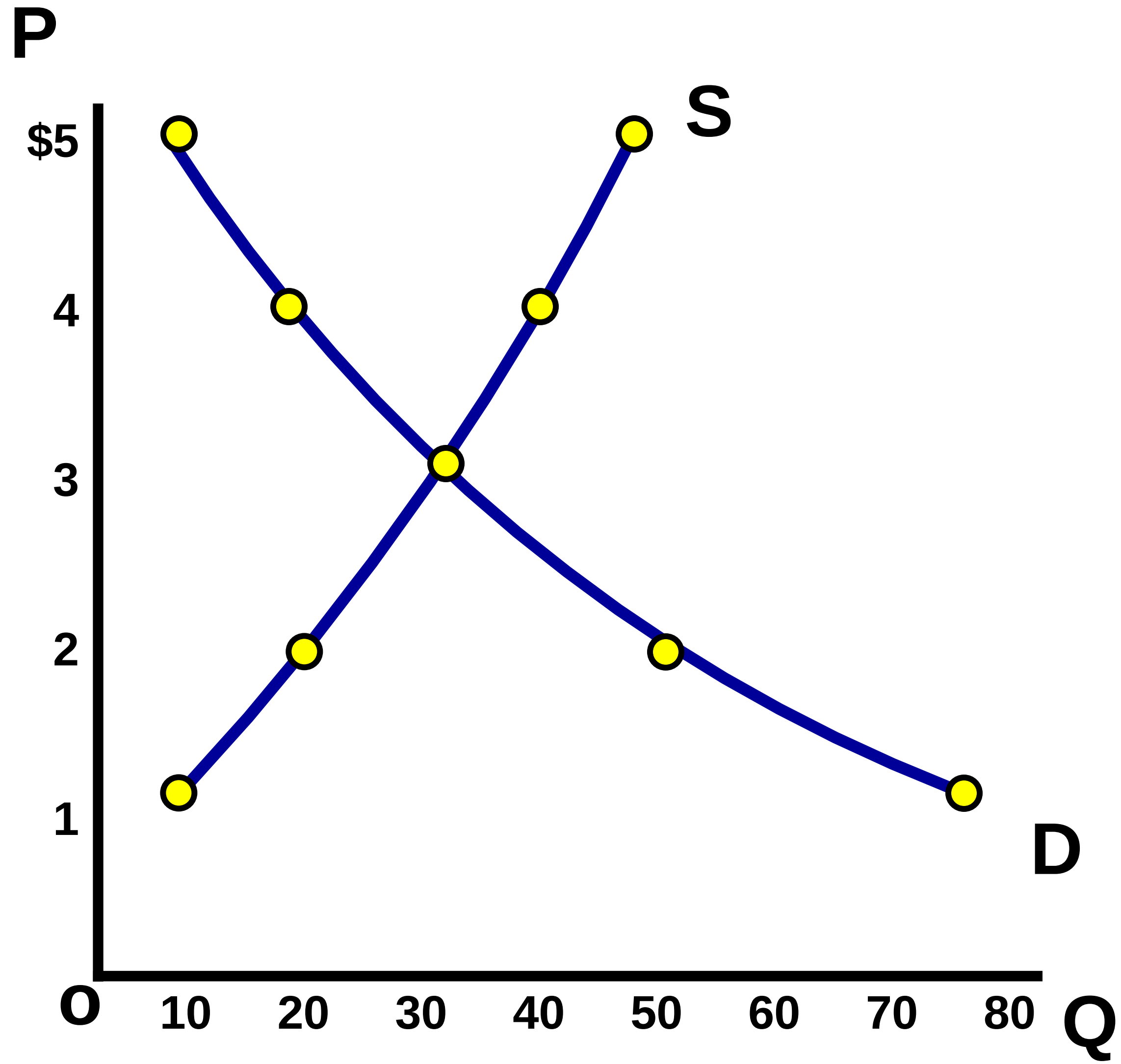
Price	Quantity Supplied
$5	~~50~~ 30
$4	~~40~~ 20
$3	~~30~~ 10
$2	~~20~~ 1
$1	~~10~~ 0



S&D together = E so What is E point on graph below?

Demand Schedule

P	Qd
$5	10
$4	20
$3	30
$2	50
$1	80

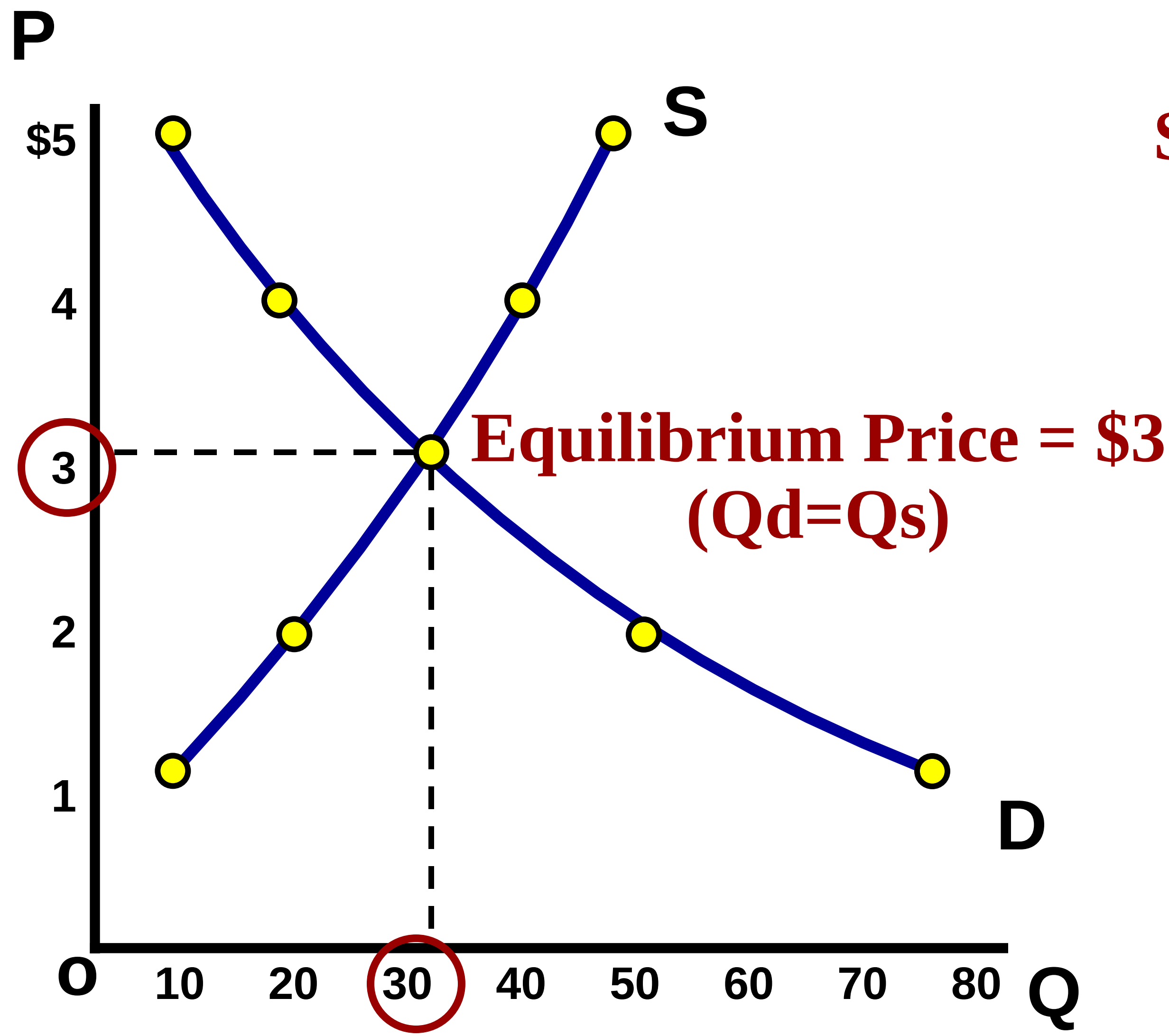


Supply Schedule

P	Qs
$5	50
$4	40
$3	30
$2	20
$1	10

Demand Schedule

P	Qd
$5	10
$4	20
$3	30
$2	50
$1	80



Equilibrium Quantity is 30

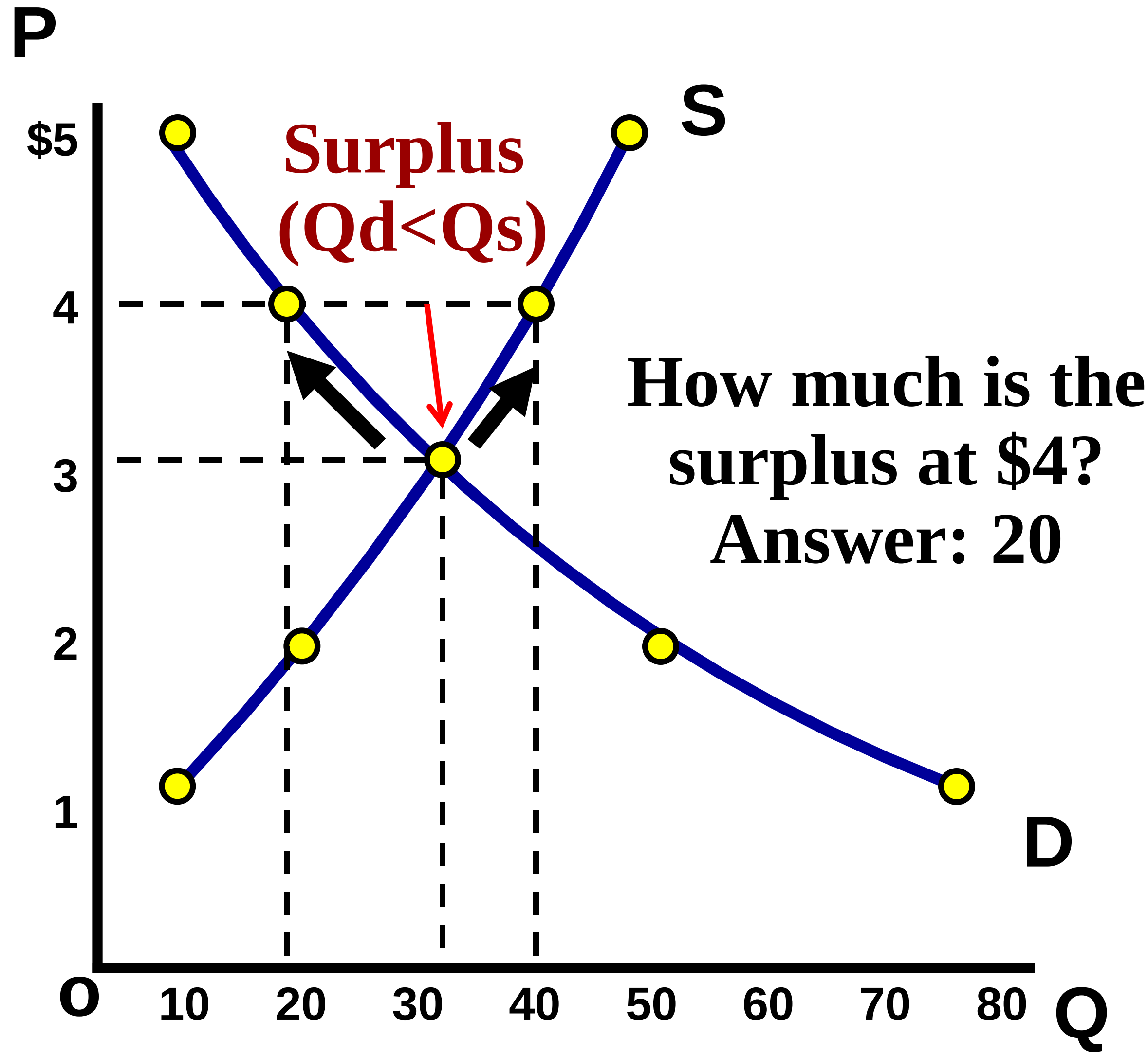
Supply Schedule

P	Qs
$5	50
$4	40
$3	30
$2	20
$1	10

At $4, there is disequilibrium. The quantity demanded is <u>less than</u> quantity supplied.

Demand Schedule

P	Qd
$5	10
$4	20
$3	30
$2	50
$1	80



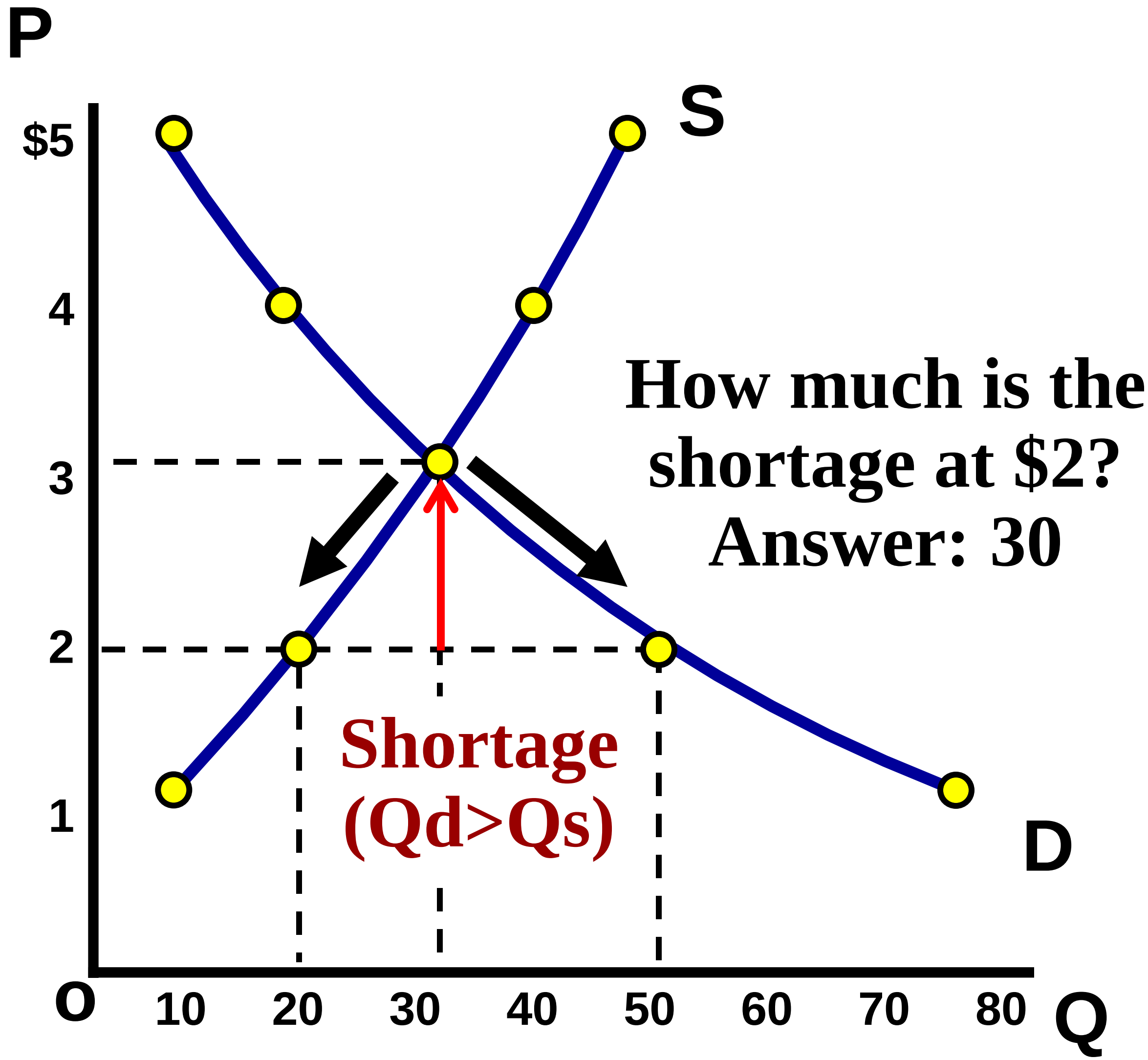
Supply Schedule

P	Qs
$5	50
$4	40
$3	30
$2	20
$1	10

At $2, there is disequilibrium. The quantity demanded is <u>greater than</u> quantity supplied.

Demand Schedule

P	Qd
$5	10
$4	20
$3	30
$2	50
$1	80



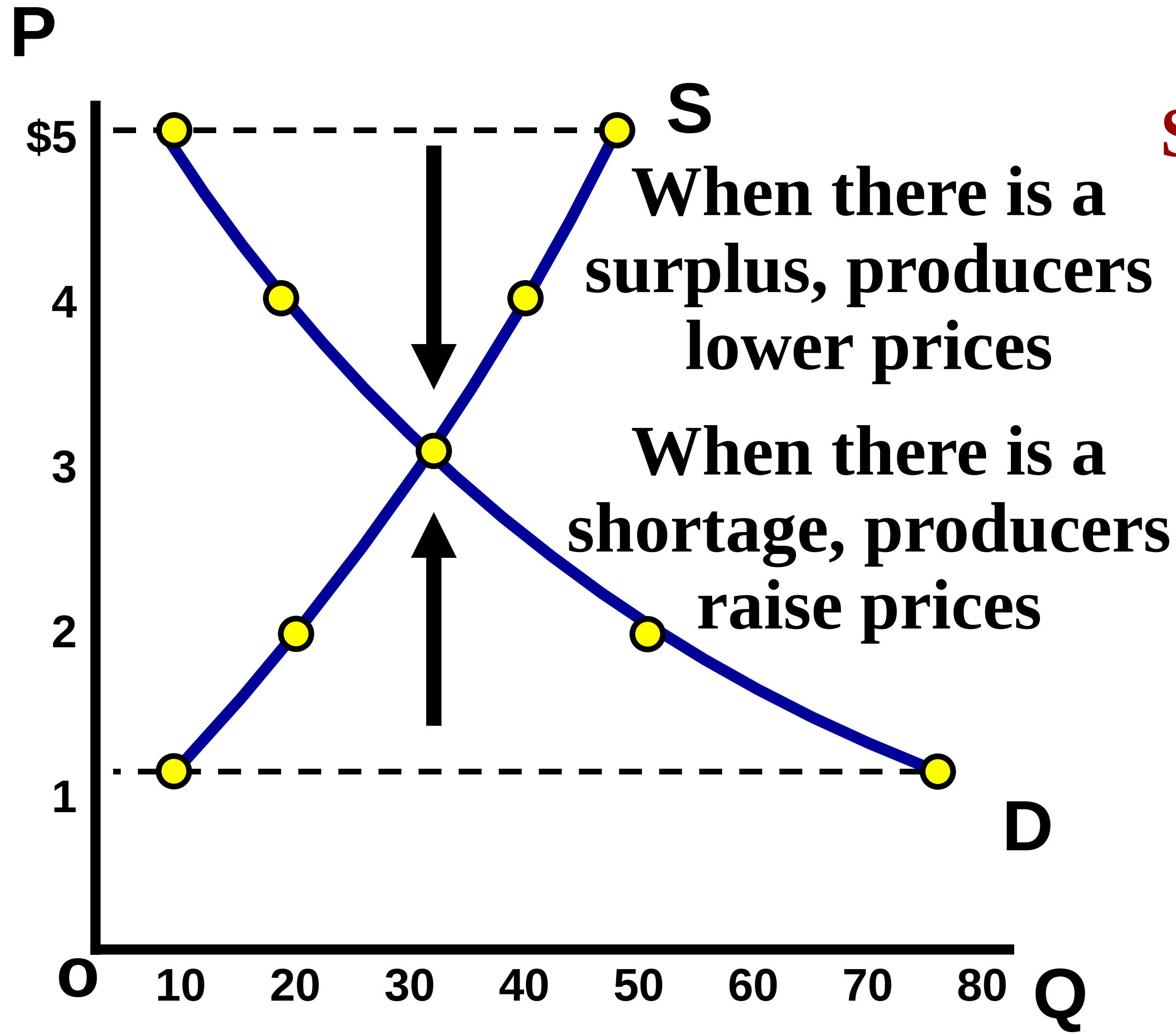
Supply Schedule

P	Qs
$5	50
$4	40
$3	30
$2	20
$1	10

The FREE MARKET system automatically pushes the price toward equilibrium.

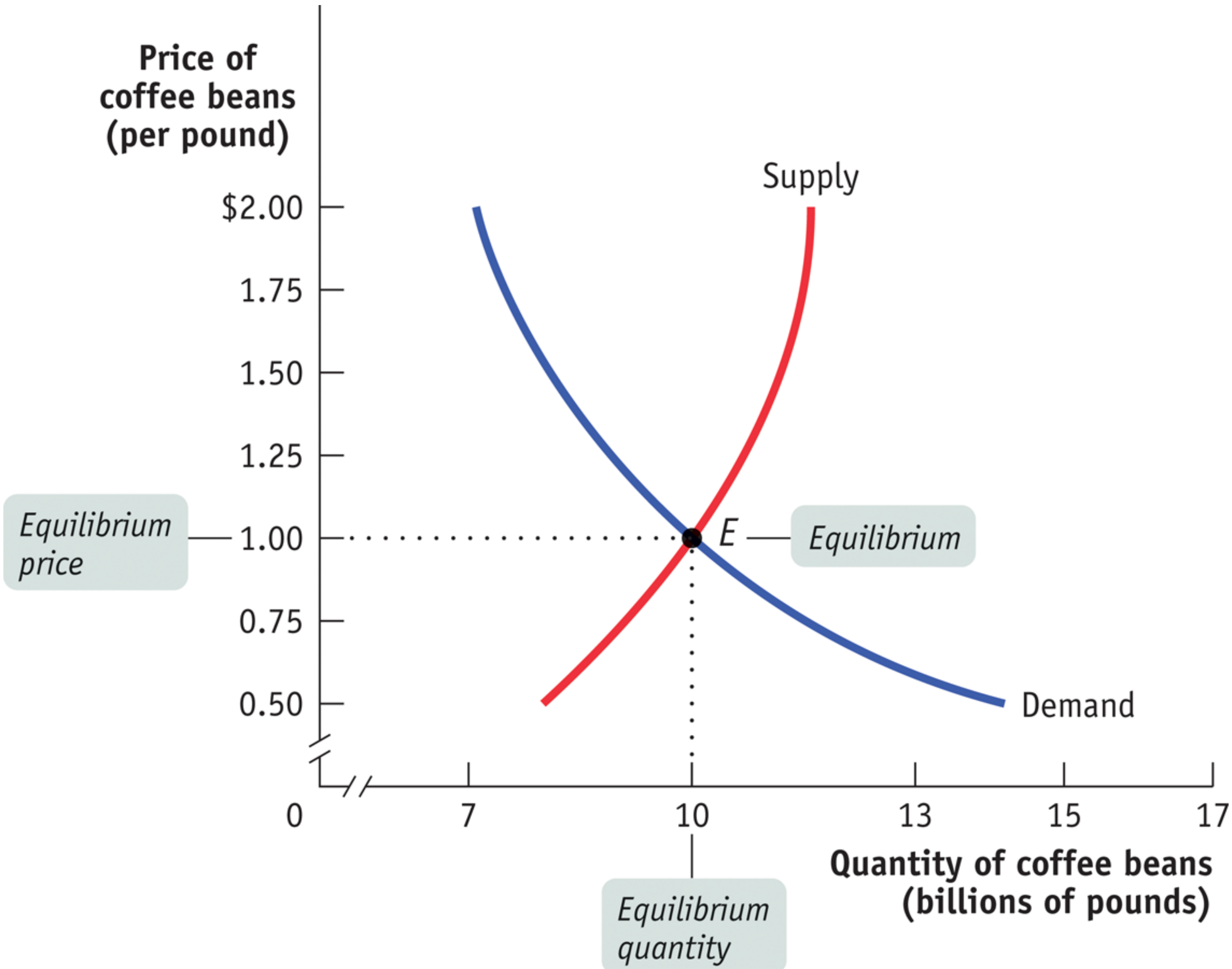
Demand Schedule

P	Qd
$5	10
$4	20
$3	30
$2	50
$1	80

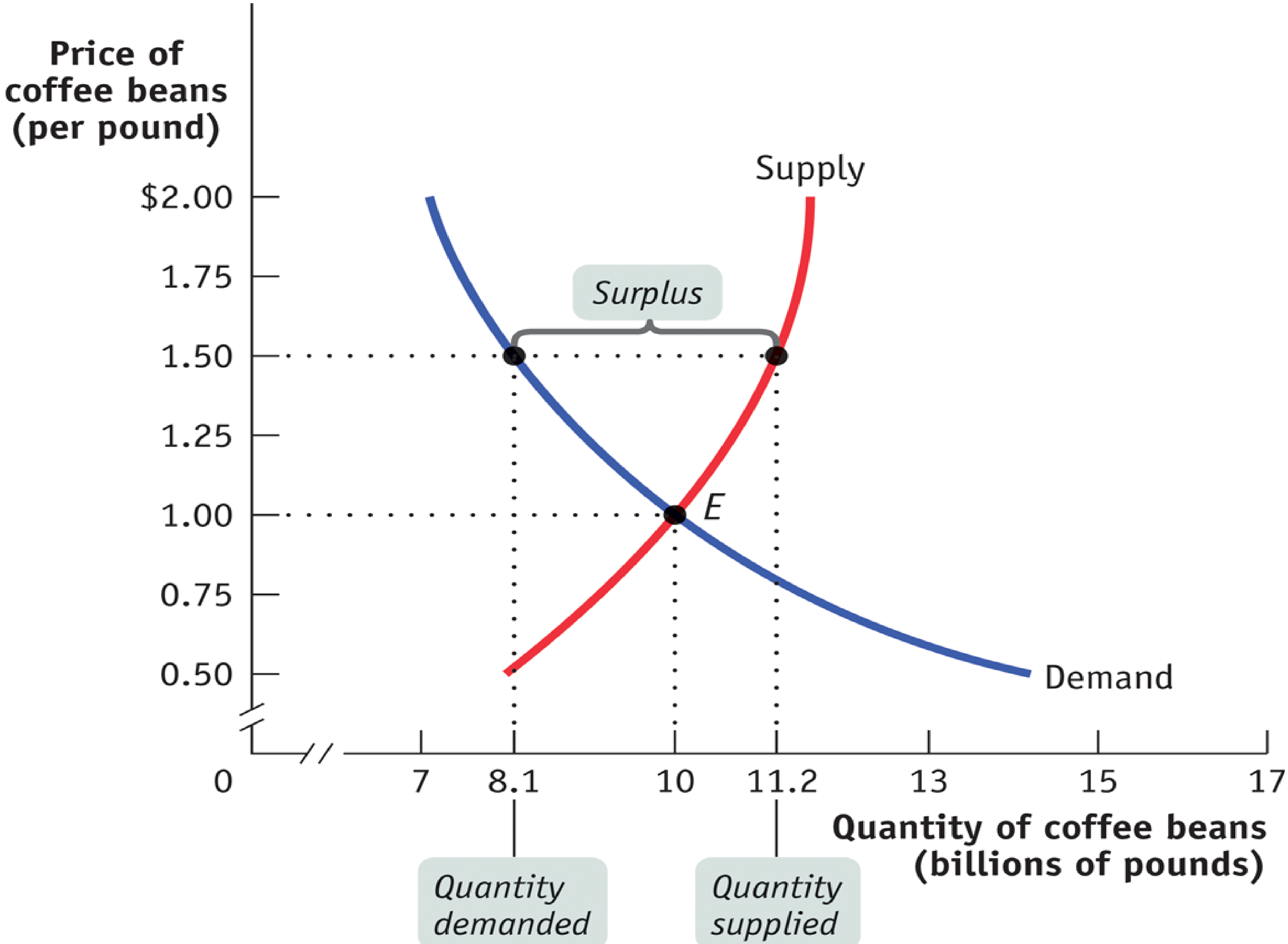


Supply Schedule

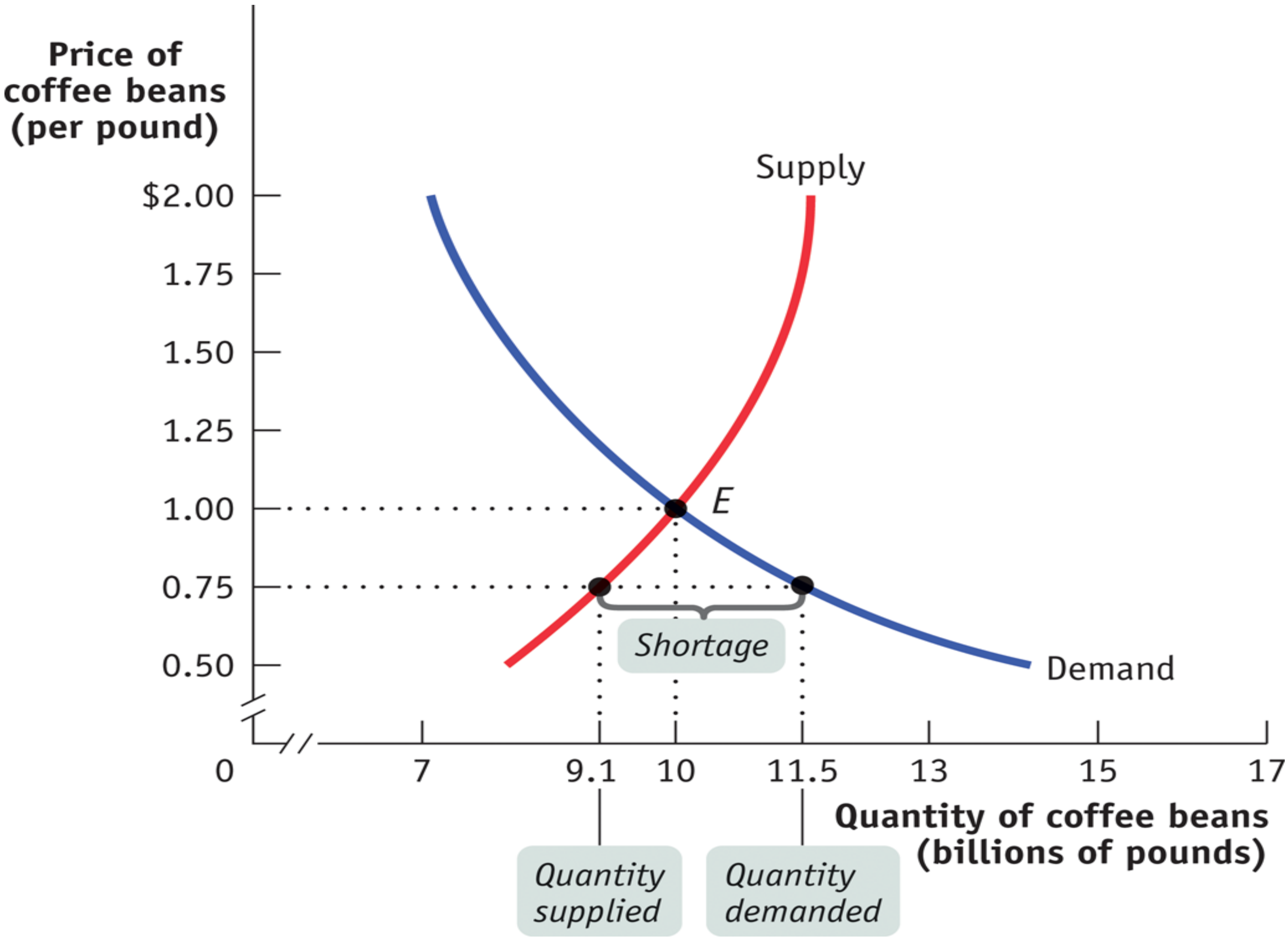
P	Qs
$5	50
$4	40
$3	30
$2	20
$1	10



ANOTHER EXAMPLE USING COFFEE TO GO WITH YOUR BURGER



A price above equilibrium creates a surplus



A price below equilibrium creates a shortage

Short Run Production

- In the short run, capital is fixed
  - Only changes in the variable labor input can change the level of output
- Short run production function

$$Q = f\,(L, \bar{K}) = f\,(L)$$

Short Run Production Costs

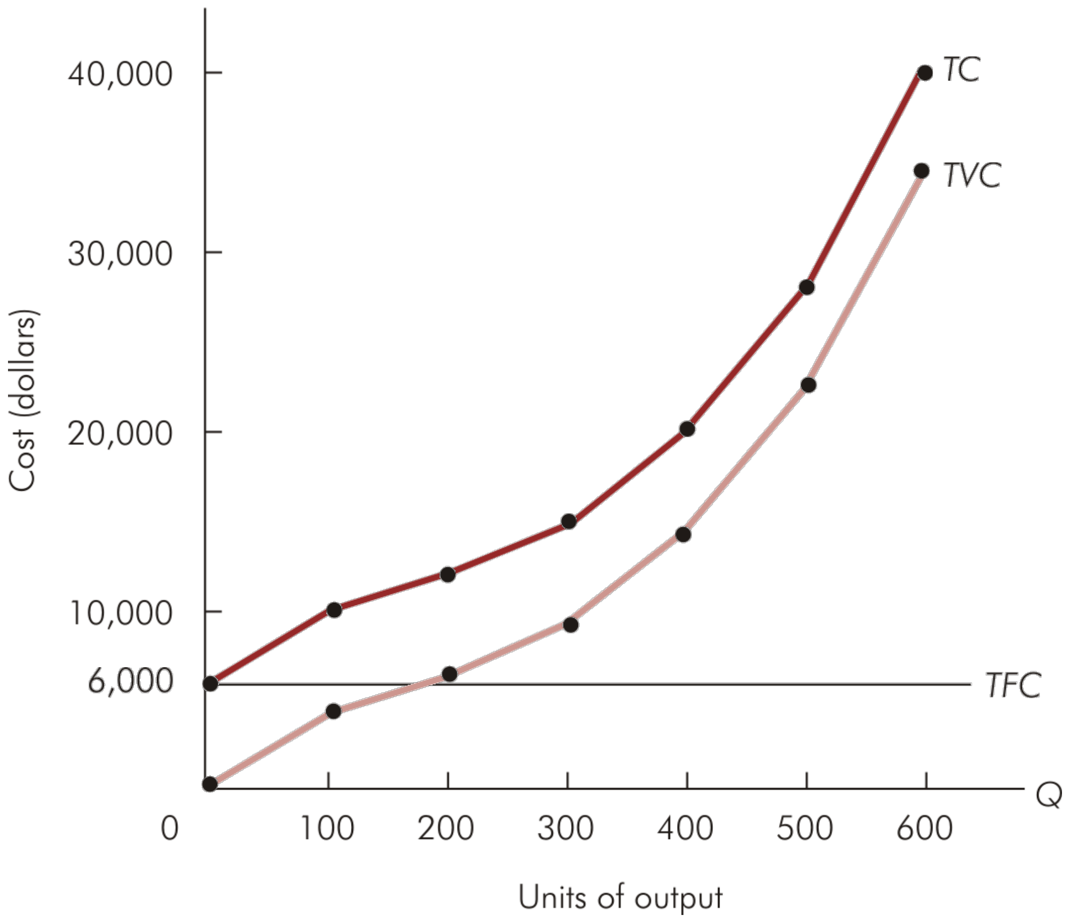
- Total fixed cost ***(TFC)***
  – Total amount paid for fixed inputs
  – Does not vary with output
- Total variable cost ***(TVC)***
  – Total amount paid for variable inputs
  – Increases as output increases
- Total cost ***(TC)***

$$TC = TFC + TVC$$

Short-Run Total Cost Schedules (Table 8.5)

Output (Q)	Total fixed cost (TFC)	Total variable cost (TVC)	Total Cost (TC=TFC+TVC)
0	$6,000	$ 0	$ 6,000
100	6,000	4,000	10,000
200	6,000	6,000	12,000
300	6,000	9,000	15,000
400	6,000	14,000	20,000
500	6,000	22,000	28,000
600	6,000	34,000	40,000

Total Cost Curves (Figure 8.3)



Average Costs

- Average fixed cost ***(AFC)***

$$AFC = \frac{TFC}{Q}$$

- Average variable cost ***(AVC)***

$$AVC = \frac{TVC}{Q}$$

- Average total cost ***(ATC)***

$$ATC = \frac{TC}{Q} = AFC + AVC$$

Short Run Marginal Cost

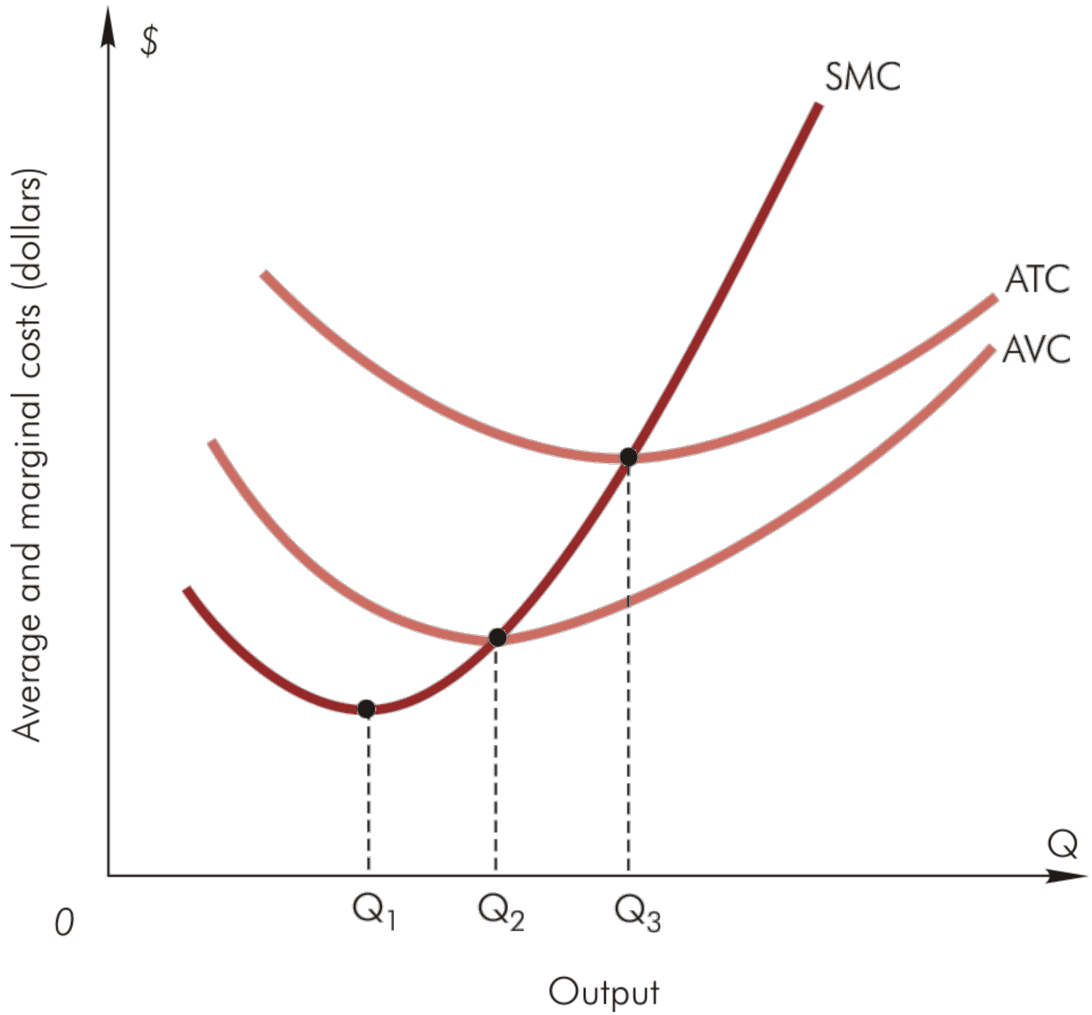
- Short run marginal cost ***(SMC)*** measures rate of change in total cost ***(TC)*** as output varies

$$SMC = \frac{\Delta TVC}{\Delta Q} = \frac{\Delta TC}{\Delta Q}$$

Average & Marginal Cost Schedules
(Table 8.6)

Output (Q)	**Average fixed cost (AFC=TFC/Q)**	**Average variable cost (AVC=TVC/Q)**	**Average total cost (ATC=TC/Q= AFC+AVC)**	**Short-run marginal cost (SMC=ΔTC/ΔQ)**
0	--	--	--	--
100	$60	$40	$100	$40
200	30	30	60	20
300	20	30	50	30
400	15	35	50	50
500	12	44	56	80
600	10	56.7	66.7	120

Short Run Average & Marginal Cost Curves (Figure 8.5)



Long-Run Costs

- Long-run total cost ***(LTC)*** for a given level of output is given by:

$$LTC = wL^* + rK^*$$

Where w & r are prices of labor & capital, respectively, & (L^*, K^*) is the input combination on the expansion path that minimizes the total cost of producing that output

Long-Run Costs

- Long-run average cost ***(LAC)*** measures the cost per unit of output when production can be adjusted so that the optimal amount of each input is employed
  - ***LAC*** is U-shaped
  - Falling ***LAC*** indicates *economies of scale*
  - Rising ***LAC*** indicates *diseconomies of scale*

$$LAC = \frac{LTC}{Q}$$

Long-Run Costs

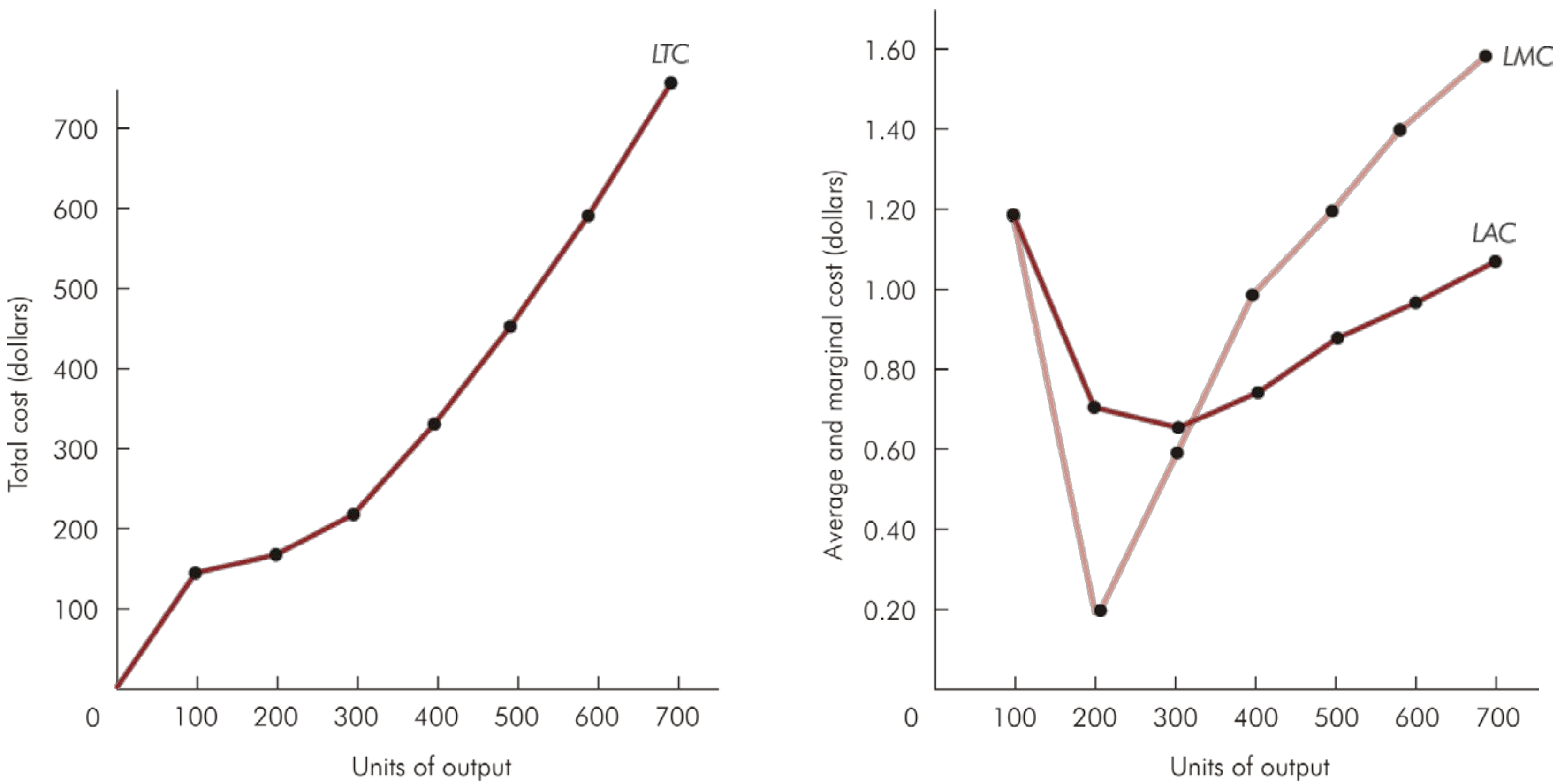
- Long-run marginal cost ***(LMC)*** measures the rate of change in long-run total cost as output changes along expansion path
  – ***LMC*** is U-shaped
  – ***LMC*** lies below ***LAC*** when ***LAC*** is falling
  – ***LMC*** lies above ***LAC*** when ***LAC*** is rising
  – ***LMC*** = ***LAC*** at the minimum value of ***LAC***

$$LMC = \frac{\Delta LTC}{\Delta Q}$$

Derivation of a Long-Run Cost Schedule (Table 9.1)

Output	Least-cost combination of		Total cost	*LAC*	*LMC*
	Labor (units)	Capital (units)	(*w* = $5, *r* = $10)		
100	10	7	$120	$1.20	$1.20
200	12	8	140	0.70	0.20
300	20	10	200	0.67	0.60
400	30	15	300	0.75	1.00
500	40	22	420	0.84	1.20
600	52	30	560	0.93	1.40
700	60	42	720	1.03	1.60

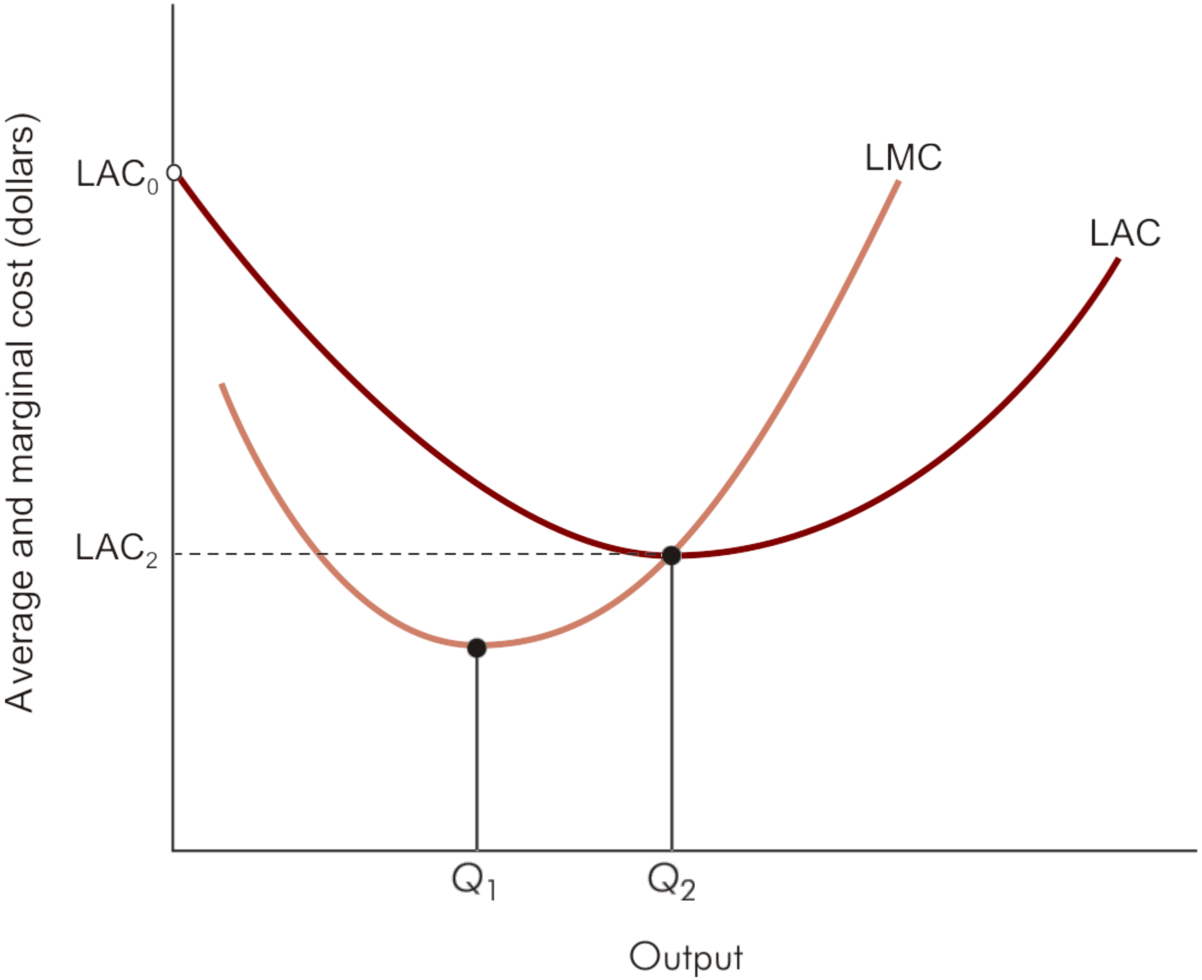
Long-Run Total, Average, & Marginal Cost (Figure 9.8)



Panel A

Panel B

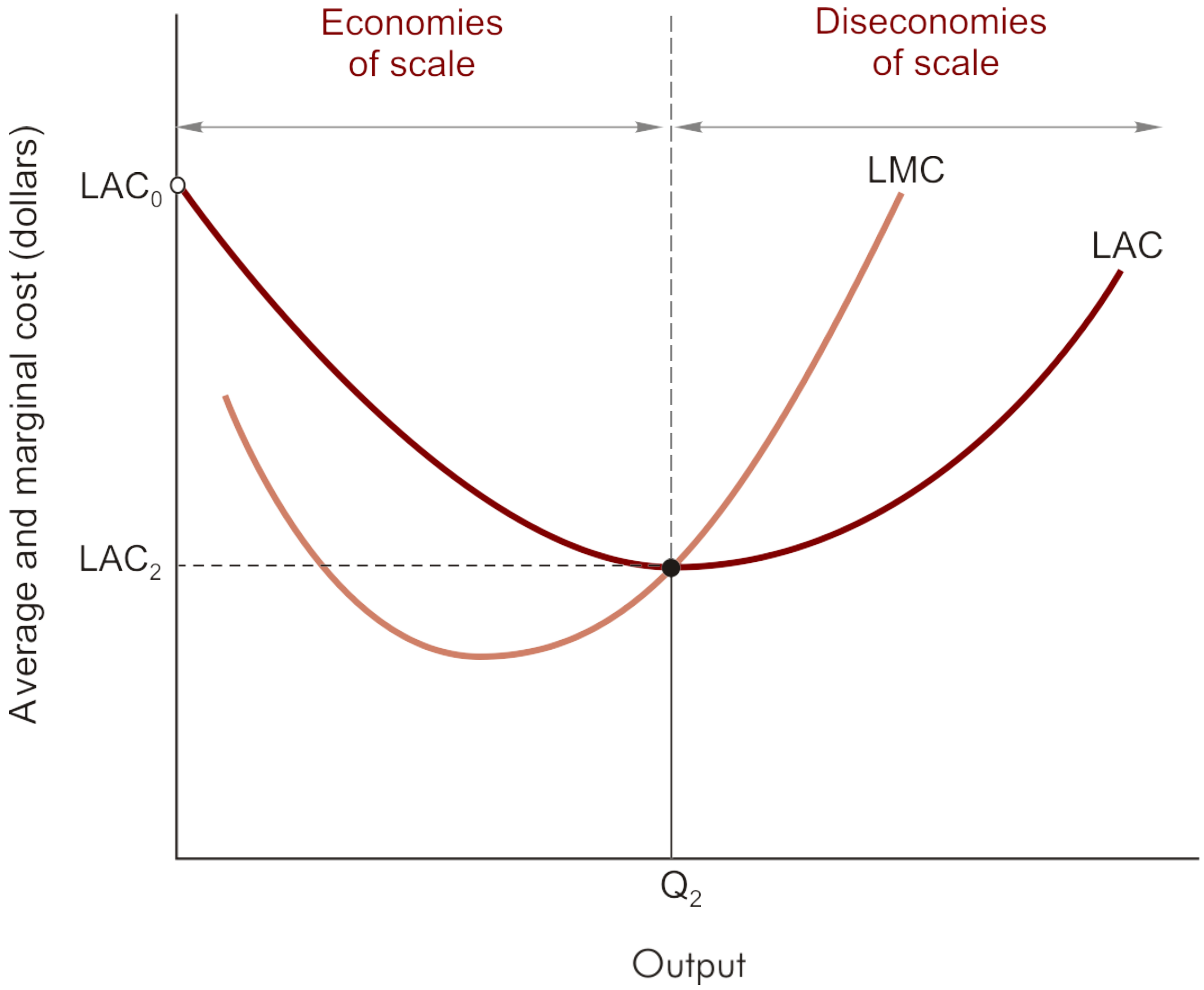
Long-Run Average & Marginal Cost Curves (Figure 9.9)



Economies of Scale

- Larger-scale firms are able to take greater advantage of opportunities for specialization & division of labor
- Scale economies also arise when quasi-fixed costs are spread over more units of output causing LAC to fall
- Variety of technological factors can also contribute to falling LAC

Economies & Diseconomies of Scale (Figure 9.10)



- **Example C1:**

Assume 100 identical firms in a perfectly **competitive** industry. Each firm faces a total cost curve of the following form:

$$C(q)=\frac{1}{300}q^3+0,2q^2+4q+10$$

(*q* denotes the firm's output and *Q* the industry's output).

(a) Find the short run supply curve for the firm *q* as a function of market price *(P)*.

(b) Find the short run supply curve for the industry.

(c) Assume that market demand is $Q=-200P+8000$. Find the price-quantity combination of the short run equilibrium.

Answer:

(a) Short-run supply curve: $P = SMC(q)$, $MC(q)=0,01q^2+0,4q+4$

$$P=SMC(q)\Rightarrow P=0,01q^2+0,4q+4\Rightarrow q=10\sqrt{P}-20$$

(b) $Q = 100q = 1000\sqrt{P} - 2000$.

(c) $Q^D = Q^S \Rightarrow -200P + 8000 = 1000\sqrt{P} - 2000 \Rightarrow P = 25 \quad Q = 3000$ and

.

For each firm:

$$q = \frac{3000}{100} = 30, \; C(q) = 400, \; AC = \frac{400}{30} = 13,33$$

and

$$\pi = TR - TC = P \cdot q - AC \cdot q = 25 \cdot 30 - 13,33 \cdot 30 = 350$$

Example C2:

Assume a perfectly competitive industry, with a large number of firms that potentially will enter. Each firm has the same cost structure, such *AC* is minimised when *q=20,* and is equal 10$ per unit. Assume that market demand is $Q = 1500 - 50P$

(a) Find the long run supply curve for the industry.

(b) Find the price-quantity combination of the long run equilibrium, the output of each firm, the number of firms and the profits of each firm.

(c) Assume that the short run total cost curve for each firm is $C(q) = 0,5q^2 - 10q + 200$

Find the short run *AC* and *MC* curves. At which level of output *AC* is minimized?

(d) Find the short run supply curve for each firm and the industry.

(e) Assume that market demand is shifting rightwards and becomes $Q = 2000 - 50P$

Find the price-quantity combination of the short run equilibrium, the output of each firm and the profits of each firm.

(f) Find the new long run equilibrium for the industry.

(a) Long run supply curve is flat for $P = LMC = LAC = 10$.

(b) $Q = 1500 - 50P = 1000$

Each firm produces *q=20*, which means that the number of firms is $\frac{1000}{20} = 50$. Also, the profits for each firm are

$$\pi = TR - TC = P \cdot q - LAC \cdot q = 10 \cdot 20 - 10 \cdot 20 = 0$$

(c) $AC = \frac{C(q)}{q} = 0,5q - 10 + \frac{200}{q}$ *and* $SMC = \frac{dC(q)}{dq} = q - 10$

AC is minimised when

$$AC = SMC \Rightarrow 0,5q - 10 + \frac{200}{q} = q - 10 \Rightarrow q = 20$$

(d) Since $P = SMC = q - 10$, we have $q = 10 + P$ for each firm

and $Q = \sum_{1}^{50} q = 50P + 500$ for industry.

(e) $Q^D = Q^S \Rightarrow 2000 - 50P = 50P + 500 \Rightarrow P = 15$ and $Q = 1250$

For each firm: $q = \frac{1250}{50} = 25$ and

$$\pi = TR - TC = P \cdot q - AC \cdot q = 15 \cdot 25 - 10{,}5 \cdot 25 = 112{,}5$$.

(f) $P = 10$ again $\Rightarrow Q = 1500$. Each firm produces 20,
the number of firms is $\frac{1500}{20} = 75$,
the profits for each firm are

$$\pi = TR - TC = P \cdot q - LAC \cdot q = 10 \cdot 20 - 10 \cdot 20 = 0$$