

①

Measor Chain Ladder:

	0	1	2	3	Input series 4 (Q1380001)	Output (3-4)
2019	$C_{10}=30$	$C_{11}=80$	$C_{12}=120$	$C_{13}=140$	140	0
2020	$C_{20}=44$	$C_{21}=110$	$C_{22}=165$	$\hat{C}_{23}=192.5055$	165	27.5055
2021	$C_{30}=65$	$C_{31}=156$		$\hat{C}_{32}=273.0078$	156	177.0078
2022	$C_{40}=65$			$\hat{C}_{43}=283.131$	65	218.131
Σwda				888.6443	526	362.6445

ΣΥΝΤΕΛΕΣΤΕΣ ΕΞΕΛΙΞΗΣ με βάση τον μέσον τιμή

$$F_1 = \frac{C_{11} + C_{21} + C_{31}}{C_{10} + C_{20} + C_{30}} = \frac{80 + 110 + 156}{30 + 44 + 65} = 2.789$$

$$F_2 = \frac{C_{12} + C_{22}}{C_{11} + C_{21}} = \frac{120 + 165}{80 + 110} = 1.5$$

$$F_3 = \frac{C_{13}}{C_{12}} = \frac{140}{120} = 1.1667$$

1/0	2/1	3/2
2.489	1.5	1.1667

ΕΚΤΙΜΩΜΕΝΕΣ ΤΕΛΙΚΕΣ ΤΙΜΕΣ

$$\hat{C}_{23} = 165 \cdot (1.1667) = 192.5055$$

$$\hat{C}_{32} = 156 \cdot (1.5) \cdot (1.1667) = 273.0078$$

$$\hat{C}_{43} = 65 \cdot (2.489) \cdot (1.5) \cdot (1.1667) = 283.131$$

$$R_2 = 192.5055 - 165 = 27.5055$$

$$\hat{R}_3 = 273.0078 - 156 = 117.0078$$

$$\hat{R}_4 = 283.131 - 65 = 218.131$$

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Μέθοδος Bornhuetter-Ferguson (B-F):

LB = 0.8

	0	1	2	3	Premium (U ^{prior}) 4	(B-F) 5 Διόρθωση	Αποτέλεσμα (4-5)
2019	C ₁₀ = 30	C ₁₁ = 80	C ₁₂ = 120	C ₁₃ = 140	190 (140)	140	0
2020	C ₂₀ = 44	C ₂₁ = 110	C ₂₂ = 165	C ₂₃ = 193.6	250 (200)	171.4	28.6
2021	C ₃₀ = 65	C ₃₁ = 156		C ₃₃ = 276.12	350 (280)	159.88	120.12
2022	C ₄₀ = 65			C ₄₃ = 293.038	370 (296)	67.962	228.038
Σύνολο					1166	539.242	376.758

$$U_2^{\text{prior}} = 250 \cdot (0.8) = 200 \quad U_3^{\text{prior}} = 350 \cdot (0.8) = 280 \quad U_4^{\text{prior}} = 370 \cdot (0.8) = 296$$

$$F_3^{\text{ult}} = \frac{1}{F_3} = \frac{1}{1.1667} = 0.857, \quad F_2^{\text{ult}} = \frac{1}{F_3 F_2} = \frac{1}{1.1667 \cdot 1.5} = 0.571, \quad F_1^{\text{ult}} = \frac{1}{F_1 F_2 F_3} = \frac{1}{1.1667 \cdot 1.5 \cdot 2.409} = 0.2296$$

NEΕΣ ΤΙΠΕΣ ΤΩΣ ΔΙΑΧΩΡΙΩΝ

① $\hat{C}_{22} = F_3^{\text{ult}} \cdot U_2^{\text{prior}} = 171.4$ ② $\hat{C}_{31} = F_2^{\text{ult}} \cdot U_3^{\text{prior}} = 159.88$ ③ $\hat{C}_{40} = F_1^{\text{ult}} \cdot U_4^{\text{prior}} = 67.96$

Αποτέλεσμα

$$\begin{aligned} \hat{R}_{2-B-F} &= 200 - 171.4 = 28.6 \\ \hat{R}_{3-B-F} &= 280 - 159.88 = 120.12 \\ \hat{R}_{4-B-F} &= 296 - 67.962 = 228.038 \end{aligned}$$

ΕΚΤΙΜΩΜΕΝΕΣ ΤΕΛΙΚΕΣ ΤΙΠΕΣ

$$\begin{aligned} \hat{C}_{23}^{B-F} &= 165 + 28.6 = 193.6 \\ \hat{C}_{33}^{B-F} &= 156 + 120.12 = 276.12 \\ \hat{C}_{43}^{B-F} &= 65 + 228.038 = 293.038 \end{aligned}$$

$$\begin{aligned} F_3^{\text{ult}} &= \frac{1}{F_3} = \frac{1}{1.1667} \\ F_2^{\text{ult}} &= \frac{1}{(1.1667)(1.5)} = 0.5714286 \\ F_1^{\text{ult}} &= \frac{1}{(1.1667)(1.5)(2.409)} \end{aligned}$$

① $\hat{C}_{22} = F_3^{\text{ult}} U_2^{\text{prior}} = \frac{1}{1.1667} (200) = 171.4281$

② $\hat{C}_{31} = F_2^{\text{ult}} U_3^{\text{prior}} = \frac{1}{(1.1667)(1.5)} (280) = 159.995$

③ $\hat{C}_{40} = F_1^{\text{ult}} U_4^{\text{prior}} = \frac{1}{(1.1667)(1.5)(2.409)} (296) = 67.9596$

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Μεθοδος Benktander-Hov, nem (B-H)

Αποδείξα:

$$\hat{R}_2^{B-H} = F_3^{ult} \hat{R}_2^{C-L} + (1 - F_3^{ult}) \hat{R}_2^{B-F} = (0.857) (27.5055) + (1 - 0.857) \cdot 28.6 = 27.662$$

$$\frac{1}{1.1667}$$

$$\hat{R}_3^{B-H} = F_2^{ult} \hat{R}_3^{C-L} + (1 - F_2^{ult}) \hat{R}_3^{B-F} = (0.571) (117.0078) + (1 - 0.571) 120.12 = 118.342$$

$$\frac{1}{4.1667(1.5)}$$

$$\hat{R}_4^{B-H} = F_1^{ult} \hat{R}_4^{C-L} + (1 - F_1^{ult}) \hat{R}_4^{B-F} = (0.2296) (218.131) + (1 - 0.2296) 228.038 = 225.713$$

$$\frac{1}{(1.1667)(1.5)(2.489)}$$

Τελικές τιμές:

$$\hat{C}_{23}^{B-H} = F_3^{ult} \hat{C}_{23}^{C-L} + (1 - F_3^{ult}) \hat{C}_{23}^{B-F} = (0.857) (192.5055) + (1 - 0.857) (193.6) = 192.662$$

$$\hat{C}_{33}^{B-H} = F_2^{ult} \hat{C}_{33}^{C-L} + (1 - F_2^{ult}) \hat{C}_{33}^{B-F} = (0.571) (273.0078) + (1 - 0.571) (276.12) = 277.1$$

$$\hat{C}_{43}^{B-H} = F_1^{ult} \hat{C}_{43}^{C-L} + (1 - F_1^{ult}) \hat{C}_{43}^{B-F} = (0.2296) (283.131) + (1 - 0.2296) (293.038) = 291.6817$$

Μεθοδος Cape-Cod (C-C):

$$L = \frac{140 + 165 + 156 + 65}{190(1.0) + 250(0.857) + 350(0.571) + 370(0.2296)} = 0.763368$$

Αποδείξα

$$\hat{R}_2 = (1 - F_3^{ult}) L \cdot P_2 = (1 - 0.857) (0.763368) 250 = 27.290$$

$$\hat{R}_3 = (1 - F_2^{ult}) L \cdot P_3 = (1 - 0.571) (0.763368) 350 = 114.6197$$

$$\hat{R}_4 = (1 - F_1^{ult}) L \cdot P_4 = (1 - 0.2296) (0.763368) 370 = 217.5765$$

Τελικές τιμές:

$$\hat{C}_{23}^{C-C} = C_{22} + R_2 = 165 + 27.290 = 192.29, \hat{C}_{43}^{C-C} = C_{40} + R_4 = 65 + 217.5965 = 282.5965$$

$$\hat{C}_{33}^{C-C} = C_{31} + R_3 = 156 + 114.6197 = 270.6197$$

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Metode Hurwicz :

	0	1	2	3	P
2019	$x_{10} = 30$	$x_{11} = 50$	$x_{12} = 40$	$x_{13} = 20$	190
2020	$x_{20} = 44$	$x_{21} = 66$	$x_{22} = 55$		250
2021	$x_{30} = 65$	$x_{31} = 91$			350
2022	$x_{40} = 65$				370

$$m_1 = \frac{x_{10} + x_{20} + x_{30} + x_{40}}{P_1 + P_2 + P_3 + P_4} = \frac{30 + 44 + 65 + 65}{190 + 250 + 350 + 370} = 0.1759$$

$$m_2 = \frac{x_{11} + x_{21} + x_{31}}{P_1 + P_2 + P_3} = \frac{50 + 66 + 91}{190 + 250 + 350} = 0.262$$

$$m_3 = \frac{x_{12} + x_{22}}{P_1 + P_2} = \frac{40 + 55}{190 + 250} = 0.2159, m_4 = \frac{x_{13}}{P_1} = 0.1053$$

$$ELR = m_1 + m_2 + m_3 + m_4 = 0.7591$$

$$p_1 = \frac{m_1 + m_2 + m_3 + m_4}{m_1 + m_2 + m_3 + m_4} = 1, p_2 = \frac{m_1 + m_2 + m_3}{m_1 + m_2 + m_3 + m_4} = \frac{0.6538}{0.7591} = 0.8613$$

$$p_3 = \frac{m_1 + m_2}{m_1 + m_2 + m_3 + m_4} = \frac{0.4379}{0.7591} = 0.5769, p_4 = \frac{m_1}{m_1 + m_2 + m_3 + m_4} = \frac{0.1759}{0.7591} = 0.2317$$

To U_j^{ind} 10×10 jika $j \geq 1$ ada :

$$U_2^{ind} = \frac{x_{20} + x_{21} + x_{22}}{P_2} = \frac{165}{0.8613} = 191.5709, U_4^{ind} = \frac{x_{40}}{P_4} = \frac{65}{0.2317} = 280.5352$$

$$U_3^{ind} = \frac{x_{30} + x_{31}}{P_3} = \frac{156}{0.5769} = 270.410$$

$$R_2^{ind} = q_2 \cdot U_2^{ind} = (1 - 0.8613) \cdot 191.5709 = 35.823$$

$$R_3^{ind} = q_3 \cdot U_3^{ind} = (1 - 0.5769) \cdot 270.410 = 114.4105$$

$$R_4^{ind} = q_4 \cdot U_4^{ind} = (1 - 0.2317) \cdot 280.5352 = 215.5352$$

$$U_2^{B-C} = P_2 \sum_{i=1}^4 m_i = 250 \cdot 0.7591 = 189.775$$

$$U_3^{B-C} = P_3 \sum_{i=1}^4 m_i = 350 \cdot 0.7591 = 265.685$$

$$U_4^{B-C} = P_4 \sum_{i=1}^4 m_i = 370 \cdot 0.7591 = 280.867$$

$$R_2^{coll} = q_2 \cdot U_2^{B-C} = (1 - 0.8613) \cdot 189.775 = 26.3218$$

$$R_3^{coll} = q_3 \cdot U_3^{B-C} = (1 - 0.5769) \cdot 265.685 = 112.411$$

$$\left. \begin{aligned} R_4^{coll} &= q_4 \cdot U_4^{B-C} \\ &= (1 - 0.2317) \cdot 280.867 = 215.79 \end{aligned} \right\}$$

$$\left. \begin{aligned} U_2^{coll} &= C_2 + R_2^{coll} \\ &= 165 + 26.3218 \end{aligned} \right\}$$

$$= 191.3218$$

$$\left. \begin{aligned} U_3^{coll} &= C_3 + R_3^{coll} = 268.411 \\ U_4^{coll} &= C_4 + R_4^{coll} = 280.79 \end{aligned} \right\}$$