Chapter 5
Cost-Volume-Profit Relationships

Solutions to Questions

5-1 The contribution margin (CM) ratio is the ratio of the total contribution margin to total sales revenue. It can also be expressed as the ratio of the contribution margin per unit to the selling price per unit. It is used in target profit and break-even analysis and can be used to quickly estimate the effect on profits of a change in sales revenue.

5-2 Incremental analysis focuses on the changes in revenues and costs that will result from a particular action.

5-3 All other things equal, Company B, with its higher fixed costs and lower variable costs, will have a higher contribution margin ratio than Company A. Therefore, it will tend to realize a larger increase in contribution margin and in profits when sales increase.

5-4 Operating leverage measures the impact on net operating income of a given percentage change in unit sales. The degree of operating leverage at a given level of sales is computed by dividing the contribution margin at that level of sales by the net operating income at that level of sales.

5-5 The break-even point is the level of sales at which profits are zero.

5-6 (a) If the selling price decreased, then the total revenue line would rise less steeply, and the break-even point would occur at a higher unit volume. (b) If the fixed cost increased, then both the fixed cost line and the total cost line would shift upward and the break-even point would occur at a higher unit volume. (c) If the variable cost per unit increased, then the total cost line would rise more steeply and the break-even point would occur at a higher unit volume.

5-7 The margin of safety is the excess of budgeted (or actual) sales over the break-even volume of sales. It is the amount by which sales can drop before losses begin to be incurred.

5-8 The sales mix is the relative proportions in which a company's products are sold. The usual assumption in cost-volume-profit analysis is that the sales mix will not change.

5-9 A higher break-even point and a lower net operating income could result if the sales mix shifted from high contribution margin products to low contribution margin products. Such a shift would cause the average contribution margin ratio in the company to decline, resulting in less total contribution margin for a given amount of sales. Thus, net operating income would decline. With a lower contribution margin ratio, the break-even point would be higher because more sales would be required to cover the same amount of fixed costs.

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The Foundational 15

1. The contribution margin per unit is calculated as follows:

Total contribution margin (a)	$8,000
Total units sold (b).....	1,000 units
Contribution margin per unit (a) ÷ (b).	$8.00 per unit

   The contribution margin per unit ($8) can also be derived by calculating the selling price per unit of $20 (= $20,000 ÷ 1,000 units) and deducting the variable expense per unit of $12 (= $12,000 ÷ 1,000 units).

2. The contribution margin ratio is calculated as follows:

Total contribution margin (a)	$8,000
Total sales (b)	$20,000
Contribution margin ratio (a) ÷ (b).....	40%

3. The variable expense ratio is calculated as follows:

Total variable expenses (a).....	$12,000
Total sales (b)	$20,000
Variable expense ratio (a) ÷ (b).....	60%

4. The increase in net operating income is calculated as follows:

Contribution margin per unit (a).....	$8.00 per unit
Increase in unit sales (b).....	1 unit
Increase in net operating income (a) × (b) ..	$8.00

5. If sales decline to 900 units, the net operating income would be computed as follows:

	Total	*Per Unit*
Sales (900 units).....	$18,000	$20.00
Variable expenses	10,800	12.00
Contribution margin.....	7,200	$ 8.00
Fixed expenses	6,000	
Net operating income ...	$ 1,200	

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The Foundational 15 (continued)

6. The new net operating income would be computed as follows:

	Total	*Per Unit*
Sales (900 units).....	$19,800	$22.00
Variable expenses	10,800	12.00
Contribution margin.....	9,000	$10.00
Fixed expenses	6,000	
Net operating income ...	$ 3,000	

7. The new net operating income would be computed as follows:

	Total	*Per Unit*
Sales (1,250 units).....	$25,000	$20.00
Variable expenses	16,250	13.00
Contribution margin.....	8,750	$ 7.00
Fixed expenses	7,500	
Net operating income ...	$ 1,250	

8. The equation method yields the break-even point in unit sales, Q, as follows:

$$\begin{aligned} \text{Profit} &= \text{Unit CM} \times Q - \text{Fixed expenses} \\ \$0 &= (\$20 - \$12) \times Q - \$6{,}000 \\ \$0 &= (\$8) \times Q - \$6{,}000 \\ \$8Q &= \$6{,}000 \\ Q &= \$6{,}000 \div \$8 \\ Q &= 750 \text{ units} \end{aligned}$$

9. The equation method yields the dollar sales to break-even as follows:

$$\begin{aligned} \text{Profit} &= \text{CM ratio} \times \text{Sales} - \text{Fixed expenses} \\ \$0 &= 0.40 \times \text{Sales} - \$6{,}000 \\ 0.40 \times \text{Sales} &= \$6{,}000 \\ \text{Sales} &= \$6{,}000 \div 0.40 \\ \text{Sales} &= \$15{,}000 \end{aligned}$$

The dollar sales to break-even ($15,000) can also be computed by multiplying the selling price per unit ($20) by the unit sales to break-even (750 units).

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The Foundational 15 (continued)

10. The equation method yields the target profit as follows:

$$\begin{aligned}\text{Profit} &= \text{Unit CM} \times Q - \text{Fixed expenses}\\ \$5{,}000 &= (\$20 - \$12) \times Q - \$6{,}000\\ \$5{,}000 &= (\$8) \times Q - \$6{,}000\\ \$8Q &= \$11{,}000\\ Q &= \$11{,}000 \div \$8\\ Q &= 1{,}375 \text{ units}\end{aligned}$$

11. The margin of safety in dollars is calculated as follows:

Sales	$20,000
Break-even sales (at 750 units)	15,000
Margin of safety (in dollars)	$ 5,000

The margin of safety as a percentage of sales is calculated as follows:

Margin of safety (in dollars) (a)	$5,000
Sales (b)	$20,000
Margin of safety percentage (a) ÷ (b)	25%

12. The degree of operating leverage is calculated as follows:

Contribution margin (a)	$8,000
Net operating income (b)	$2,000
Degree of operating leverage (a) ÷ (b)	4.0

13. A 5% increase in unit sales should result in a 20% increase in net operating income, computed as follows:

Degree of operating leverage (a)	4.0
Percent increase in sales (b)	5%
Percent increase in net operating income (a) × (b)	20%

14. The degree of operating leverage is calculated as follows:

Contribution margin ($20,000 – $6,000) (a)	$14,000
Net operating income (b)	$2,000
Degree of operating leverage (a) ÷ (b)	7.0

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The Foundational 15 (continued)

15. A 5% increase in unit sales should result in a 35% increase in net operating income, computed as follows:

Degree of operating leverage (a)	7.0
Percent increase in sales (b).....	5%
Percent increase in net operating income (a) × (b) ...	35%

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Problem 5-19 (45 minutes)

1.

Sales (15,000 units × $70 per unit)	$1,050,000
Variable expenses (15,000 units × $40 per unit)	600,000
Contribution margin	450,000
Fixed expenses	540,000
Net operating loss	$ (90,000)

2. $$\text{Unit sales to break even} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$540{,}000}{\$70 \text{ per unit} - \$40 \text{ per unit}}$$

$$= 18{,}000 \text{ units}$$

18,000 units × $70 per unit = $1,260,000 to break even

3. See the next page.

4. At a selling price of $58 per unit, the contribution margin is $18 per unit. Therefore:

$$\text{Unit sales to break even} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$540{,}000}{\$18}$$

$$= 30{,}000 \text{ units}$$

30,000 units × $58 per unit = $1,740,000 to break even.

This break-even point is different from the break-even point in part (2) because of the change in selling price. With the change in selling price, the unit contribution margin drops from $30 to $18, resulting in an increase in the break-even point.

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Problem 5-19 (continued)

3.

Unit Selling Price	*Unit Variable Expense*	*Unit Contribution Margin*	*Volume (Units)*	*Total Contribution Margin*	*Fixed Expenses*	*Net operating income (loss)*
$70	$40	$30	15,000	$450,000	$540,000	$ (90,000)
$68	$40	$28	20,000	$560,000	$540,000	$ 20,000
$66	$40	$26	25,000	$650,000	$540,000	$110,000
$64	$40	$24	30,000	$720,000	$540,000	$180,000
$62	$40	$22	35,000	$770,000	$540,000	$230,000
$60	$40	$20	40,000	$800,000	$540,000	$260,000
$58	$40	$18	45,000	$810,000	$540,000	$270,000
$56	$40	$16	50,000	$800,000	$540,000	$260,000

The maximum profit is $270,000. This level of profit can be earned by selling 45,000 units at a price of $58 each.

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Problem 5-20 (75 minutes)

1. a.

Selling price	$25	100%
Variable expenses.....	15	60%
Contribution margin	$10	40%

$$\text{Profit} = \text{Unit CM} \times Q - \text{Fixed expenses}$$
$$\$0 = \$10 \times Q - \$210{,}000$$
$$\$10Q = \$210{,}000$$
$$Q = \$210{,}000 \div \$10$$
$$Q = 21{,}000 \text{ balls}$$

Alternative solution:

$$\text{Unit sales to break even} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$210{,}000}{\$10}$$

$$= 21{,}000 \text{ balls}$$

b. The degree of operating leverage is:

$$\text{Degree of operating leverage} = \frac{\text{Contribution margin}}{\text{Net operating income}}$$

$$= \frac{\$300{,}000}{\$90{,}000} = 3.33 \text{ (rounded)}$$

2. The new CM ratio will be:

Selling price.....	$25	100%
Variable expenses.....	18	72%
Contribution margin	$ 7	28%

The new break-even point will be:

$$\text{Profit} = \text{Unit CM} \times Q - \text{Fixed expenses}$$
$$\$0 = \$7 \times Q - \$210{,}000$$
$$\$7Q = \$210{,}000$$
$$Q = \$210{,}000 \div \$7$$
$$Q = 30{,}000 \text{ balls}$$

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Problem 5-20 (continued)

Alternative solution:

$$\text{Unit sales to break even} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$210{,}000}{\$7}$$

$$= 30{,}000 \text{ balls}$$

3. $$\text{Profit} = \text{Unit CM} \times Q - \text{Fixed expenses}$$
$$\$90{,}000 = \$7 \times Q - \$210{,}000$$
$$\$7Q = \$90{,}000 + \$210{,}000$$
$$Q = \$300{,}000 \div \$7$$
$$Q = 42{,}857 \text{ balls (rounded)}$$

Alternative solution:

$$\text{Unit sales to attain target profit} = \frac{\text{Target profit + Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$90{,}000 + \$210{,}000}{\$7} = 42{,}857 \text{ balls}$$

Thus, sales will have to increase by 12,857 balls (= 42,857 balls – 30,000 balls = 12,857 balls) to earn the same amount of net operating income as last year. The computations above and in part (2) show the dramatic effect that increases in variable costs can have on an organization. The effects on Northwood Company are summarized below:

	Present	*Expected*
Break-even point (in balls).....	21,000	30,000
Sales (in balls) needed to earn a $90,000 profit ..	30,000	42,857

Note that if variable costs do increase next year, then the company will just break even if it sells the same number of balls (30,000) as it did last year.

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Problem 5-20 (continued)

4. The contribution margin ratio last year was 40%. If we let P equal the new selling price, then:

$$\begin{aligned} P &= \$18 + 0.40P \\ 0.60P &= \$18 \\ P &= \$18 \div 0.60 \\ P &= \$30 \end{aligned}$$

To verify:

Selling price	$30	100%
Variable expenses	18	60%
Contribution margin.....	$12	40%

Therefore, to maintain a 40% CM ratio, a $3 increase in variable costs would require a $5 increase in the selling price.

5. The new CM ratio would be:

Selling price.....	$25	100%
Variable expenses	9*	36%
Contribution margin	$16	64%

*$15 – ($15 × 40%) = $9

The new break-even point would be:

$$\begin{aligned} \text{Profit} &= \text{Unit CM} \times Q - \text{Fixed expenses} \\ \$0 &= \$16 \times Q - (\$210{,}000 \times 2) \\ \$16Q &= \$420{,}000 \\ Q &= \$420{,}000 \div \$16 \\ Q &= 26{,}250 \text{ balls} \end{aligned}$$

Alternative solution:

$$\text{Unit sales to break even} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$420{,}000}{\$16} = 26{,}250 \text{ balls}$$

Although this new break-even point is greater than the company's present break-even point of 21,000 balls [see Part (1) above], it is less than the break-even point will be if the company does not automate and variable labor costs rise next year [see Part (2) above].

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Problem 5-20 (continued)

6. a. Profit = Unit CM × Q − Fixed expenses
$\$90{,}000 = \$16 \times Q - \$420{,}000$
$\$16Q = \$90{,}000 + \$420{,}000$
$Q = \$510{,}000 \div \$16$
$Q = 31{,}875$ balls

Alternative solution:

$$\text{Unit sales to attain target profit} = \frac{\text{Target profit + Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$90{,}000 + \$420{,}000}{\$16}$$

$$= 31{,}875 \text{ balls}$$

Thus, the company will have to sell 1,875 more balls (31,875 − 30,000 = 1,875) than now being sold to earn a profit of $90,000 per year. However, this is still less than the 42,857 balls that would have to be sold to earn a $90,000 profit if the plant is not automated and variable labor costs rise next year [see Part (3) above].

b. The contribution income statement would be:

Sales (30,000 balls × $25 per ball)	$750,000
Variable expenses (30,000 balls × $9 per ball)	270,000
Contribution margin	480,000
Fixed expenses	420,000
Net operating income	$ 60,000

$$\text{Degree of operating leverage} = \frac{\text{Contribution margin}}{\text{Net operating income}}$$

$$= \frac{\$480{,}000}{\$60{,}000} = 8$$

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Problem 5-20 (continued)

c. This problem illustrates the difficulty faced by some companies. When variable labor costs increase, it is often difficult to pass these cost increases along to customers in the form of higher prices. Thus, companies are forced to automate resulting in higher operating leverage, often a higher break-even point, and greater risk for the company.

   There is no clear answer as to whether one should have been in favor of constructing the new plant.

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Problem 5-21 (30 minutes)

1.

	Product							
	White		*Fragrant*		*Loonzain*		*Total*	
Percentage of total sales	40%		24%		36%		100%	
Sales.....	$300,000	100%	$180,000	100%	$270,000	100%	$750,000	100%
Variable expenses.....	216,000	72%	36,000	20%	108,000	40%	360,000	48%
Contribution margin ..	$ 84,000	28%	$144,000	80%	$162,000	60%	390,000	52% *
Fixed expenses.....							449,280	
Net operating income (loss)							$ (59,280)	

*$390,000 ÷ $750,000 = 52%

2. Break-even sales would be:

$$\text{Dollar sales to break even} = \frac{\text{Fixed expenses}}{\text{CM ratio}}$$

$$= \frac{\$449,280}{0.52} = \$864,000$$

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Problem 5-21 (continued)

3. Memo to the president:

   Although the company met its sales budget of $750,000 for the month, the mix of products changed substantially from that budgeted. This is the reason the budgeted net operating income was not met, and the reason the break-even sales were greater than budgeted. The company's sales mix was planned at 20% White, 52% Fragrant, and 28% Loonzain. The actual sales mix was 40% White, 24% Fragrant, and 36% Loonzain.

   As shown by these data, sales shifted away from Fragrant Rice, which provides our greatest contribution per dollar of sales, and shifted toward White Rice, which provides our least contribution per dollar of sales. Although the company met its budgeted level of sales, these sales provided considerably less contribution margin than we had planned, with a resulting decrease in net operating income. Notice from the attached statements that the company's overall CM ratio was only 52%, as compared to a planned CM ratio of 64%. This also explains why the break-even point was higher than planned. With less average contribution margin per dollar of sales, a greater level of sales had to be achieved to provide sufficient contribution margin to cover fixed costs.

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Problem 5-22 (60 minutes)

1. The CM ratio is 30%.

	Total	*Per Unit*	*Percent of Sales*
Sales (19,500 units).....	$585,000	$30.00	100%
Variable expenses.....	409,500	21.00	70%
Contribution margin	$175,500	$ 9.00	30%

The break-even point is:

$$\begin{aligned}
\text{Profit} &= \text{Unit CM} \times Q - \text{Fixed expenses} \\
\$0 &= (\$30 - \$21) \times Q - \$180{,}000 \\
\$0 &= (\$9) \times Q - \$180{,}000 \\
\$9Q &= \$180{,}000 \\
Q &= \$180{,}000 \div \$9 \\
Q &= 20{,}000 \text{ units}
\end{aligned}$$

20,000 units × $30 per unit = $600,000 in sales

Alternative solution:

$$\text{Unit sales to break even} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$180{,}000}{\$9.00} = 20{,}000 \text{ units}$$

$$\text{Dollar sales to break even} = \frac{\text{Fixed expenses}}{\text{CM ratio}}$$

$$= \frac{\$180{,}000}{0.30} = \$600{,}000 \text{ in sales}$$

2. Incremental contribution margin:

$80,000 increased sales × 0.30 CM ratio.....	$24,000
Less increased advertising cost	16,000
Increase in monthly net operating income	$ 8,000

Since the company is now showing a loss of $4,500 per month, if the changes are adopted, the loss will turn into a profit of $3,500 each month ($8,000 – $4,500 = $3,500).

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Problem 5-22 (continued)

3. Sales (39,000 units @ $27.00 per unit*)..... $1,053,000
   Variable expenses
   (39,000 units @ $21.00 per unit)..... 819,000
   Contribution margin 234,000
   Fixed expenses ($180,000 + $60,000) 240,000
   Net operating loss..... $ (6,000)

   *$30.00 – ($30.00 × 0.10) = $27.00

4. Profit = Unit CM × Q – Fixed expenses
   $9,750 = ($30.00 – $21.75) × Q – $180,000
   $9,750 = ($8.25) × Q – $180,000
   $8.25Q = $189,750
   Q = $189,750 ÷ $8.25
   Q = 23,000 units

   *$21.00 + $0.75 = $21.75

   Alternative solution:

$$\text{Unit sales to attain target profit} = \frac{\text{Target profit + Fixed expenses}}{\text{CM per unit}}$$

$$= \frac{\$9{,}750 + \$180{,}000}{\$8.25^{**}}$$

$$= 23{,}000 \text{ units}$$

   **$30.00 – $21.75 = $8.25

5. a. The new CM ratio would be:

	Per Unit	*Percent of Sales*
Sales.....	$30.00	100%
Variable expenses	18.00	60%
Contribution margin.....	$12.00	40%

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Problem 5-22 (continued)

The new break-even point would be:

$$\text{Unit sales to break even} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$180{,}000 + \$72{,}000}{\$12.00}$$

$$= 21{,}000 \text{ units}$$

$$\text{Dollar sales to break even} = \frac{\text{Fixed expenses}}{\text{CM ratio}}$$

$$= \frac{\$180{,}000 + \$72{,}000}{0.40}$$

$$= \$630{,}000$$

b. Comparative income statements follow:

	Not Automated			*Automated*		
	Total	*Per Unit*	*%*	*Total*	*Per Unit*	*%*
Sales (26,000 units)	$780,000	$30.00	100	$780,000	$30.00	100
Variable expenses	546,000	21.00	70	468,000	18.00	60
Contribution margin	234,000	$ 9.00	30	312,000	$12.00	40
Fixed expenses	180,000			252,000		
Net operating income	$ 54,000			$ 60,000		

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Problem 5-22 (continued)

c. Whether or not the company should automate its operations depends on how much risk the company is willing to take and on prospects for future sales. The proposed changes would increase the company's fixed costs and its break-even point. However, the changes would also increase the company's CM ratio (from 0.30 to 0.40). The higher CM ratio means that once the break-even point is reached, profits will increase more rapidly than at present. If 26,000 units are sold next month, for example, the higher CM ratio will generate $6,000 (= $60,000 – $54,000) more in profits than if no changes are made.

The greatest risk of automating is that future sales may drop back down to present levels (only 19,500 units per month), and as a result, losses will be even larger than at present due to the company's greater fixed costs. (Note the problem states that sales are erratic from month to month.) In sum, the proposed changes will help the company if sales continue to trend upward in future months; the changes will hurt the company if sales drop back down to or near present levels.

Note to the Instructor: Although it is not asked for in the problem, if time permits you may want to compute the point of indifference between the two alternatives in terms of units sold; i.e., the point where profits will be the same under either alternative. At this point, total revenue will be the same; hence, we include only costs in our equation:

$$\begin{aligned} \text{Let } Q &= \text{Point of indifference in units sold} \\ \$21.00Q + \$180{,}000 &= \$18.00Q + \$252{,}000 \\ \$3.00Q &= \$72{,}000 \\ Q &= \$72{,}000 \div \$3.00 \\ Q &= 24{,}000 \text{ units} \end{aligned}$$

If more than 24,000 units are sold in a month, the proposed plan will yield the greater profits; if less than 24,000 units are sold in a month, the present plan will yield the greater profits (or the least loss).

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