Exercise 1

Consider a homogeneous product market with an inverse demand function $P = 90 - 2Q$, where P is the product price and Q is total quantity demanded. The market is served by an incumbent firm 1 with cost function $C = 30q_1$. Currently, firm 2 is contemplating entry. Firm 2 has the same marginal cost as firm 1, but it also needs to pay a fixed entry cost F that is sunk upon entry. Hence, its cost function is $C_2 = F + 30q_2$. Let $F = 100$. Accommodation of entry by the incumbent ($i.e.$, a passive reaction from the incumbent) implies that quantity competition will ensue. However, in case of entry, firm 1 threatens to produce the competitive output, so that $P = MC$.

a) Find the monopoly profit in the case of no entry

b) Find both firms' profit in the case of entry accommodation

c) Find both firms' profit in the case of entry and aggressive reaction by the incumbent

d) Construct the game tree assuming that in case of entry, firm 2 enters producing the Cournot-equilibrium quantity. Find the subgame-perfect Nash equilibrium and explain your thought in no more than six lines

Indicative answer

a) In case of non-entry, the monopoly profit is: $\pi_1^M = (90 - 2q_1)q_1 - 30q_1$.

   Maximizing the above yields: $q_1^M = 15,\ P^M = 60,\ \pi_1^M = 450$.

b) To find the Cournot equilibrium that follows entry and passive reaction, let us start with the incumbent, who's profit function is: $\pi_1 = [90 - 2(q_1 + q_2)]q_1 - 30q_1$.

   Maximizing the above we get the incumbent's reaction function:

$$\frac{\partial \pi_1}{\partial q_1} = 90 - 2q_1 - q_2 - 30 = 0 \Leftrightarrow q_1 = \frac{90 - 30}{2 \times 2} - \frac{1}{2}q_2 = 15 - \frac{1}{2}q_2.$$

   A symmetric reaction function holds for firm B: $q_2 = 15 - \frac{1}{2}q_1$.

   Solving the two equations system we obtain: $q_A^C = q_B^C = 10$.

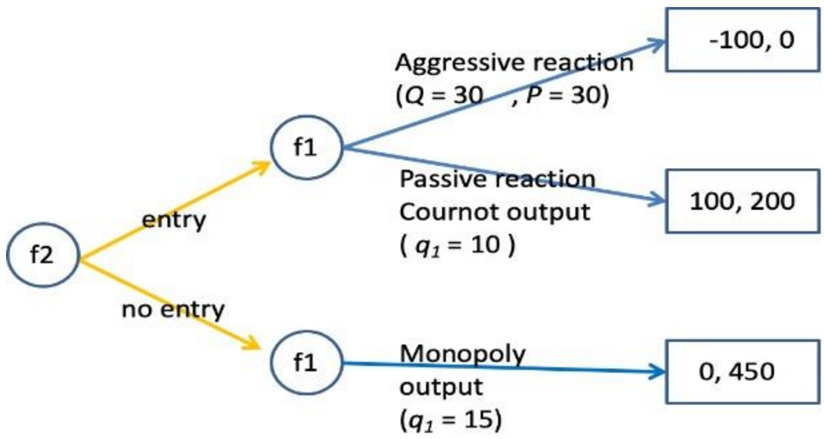
   Replacing $Q^C = q_A^C + q_B^C = 20$ in the demand function we obtain the equilibrium price: $p^C = 50$.

   Cournot-equilibrium profits are: $\pi_1 = (50 - 30) \times 10 = 200,\ \pi_2 = (50 - 30) \times 10 - 100 = 100$.

c) In case of aggressive reaction we consider that the incumbent produces as much output as it is needed to drive the price equal to marginal cost. Hence, total output in the market is found by solving $30 = 90 - 2Q \Leftrightarrow Q = 30$.

   The above is total output and it is irrelevant how it is divided between the two firms, since at the end both firms are making no gross profit, so the net profit of the incumbent is zero while that of the entrant -100, equal to its fixed cost which cannot be covered.

d) The game tree is the following



The incumbent's threat to play aggressively is non-credible since by playing passively she gets a profit of 200, while by playing aggressively she gets zero profit. Hence, the entrant chooses the best for him between entering and staying out knowing that in case of entry his profit will be that of Cournot equilibrium, *i.e.*, 100. Obviously, the subgame-perfect Nash equilibrium is (Entry, Accommodation).

Exercise 2

A picturesque village has 4,000 inhabitants. Moreover, 6,000 tourists visit the village each year, from June to August. There are two taverns in this village that open during the tourist season only. The two taverns must decide the price for the "menu of the day" per person for the current year so as to print the menus and arrange their supplies accordingly. Everything has to be arranged on time so that both taverns to open one day in advance (May 31st) in order to welcome their first customers on the first of June. The owners of the two taverns must decide simultaneously and irrevocably the price for the "menu of the day" for the current season. Each tavern's owner (who, at the same time, is the tavern's manager) can choose between a low price of €8, a medium price of €10, or a high price of €12. All natives become fully informed about both prices and go always to the cheapest tavern. If the two taverns charge the same price then assume that 50% of the local population goes to one tavern and the other 50% to the other. Tourists are uninformed about prices and pick one tavern randomly (each has 50% probability to be selected by a tourist). [*Hint*: Assuming that each tourist stays for all the tourist season in the village and that each village inhabitant goes to a tavern each day, do your analysis for a single day].

1. Represent the game in a matrix form and find its unique Nash Equilibrium. What price each tavern's owner is expected to set?

2. According to a new law, taverns are not allowed to price a daily menu below €9. Moreover, each tavern's owner thinks about a new pricing policy: "*matching the rival's price if it is lower than mine*". This pricing policy is legally binding once announced.

3. Represent the game after the aforementioned law in a matrix form before and after both taverns' owners follow the "matching the rival's price if it is lower than mine" pricing policy. What price each tavern's owner is expected to set in equilibrium in each case?

4. Discuss the effectiveness of this pricing policy on taverns' profits.

Indicative answer

1. Each tavern's owner (To) constitutes a player. Hence, we have players To1 and To2
Each tavern's owner must choose one among three prices (P) (Low: €8; Medium: €10; High: €12).
<u>Payoffs</u>: The revenues (R) to each player (Ri and Rj) at the end of the game depend on his chosen price and the price chosen by the rival tavern's owner.
<u>Toi chooses P=€8 and Toj chooses P=€8⇒</u>

- Natives (4,000): Since the two taverns have equal price, the natives are indifferent between the two taverns. We assume that they will be shared between the two taverns.
- Tourists (6,000): Each tavern has 50% probability to be selected by a tourist. Hence, they will be shared between the two taverns.
- Ri=2,000 x €8 + 3,000 x €8 = €40,000
- Rj=2,000 x €8 + 3,000 x €8 = €40,000

<u>Toi chooses P=€8 and Toj chooses P=€10⇒</u>

Natives go to Tavern i because this is the cheapest. Tourists are shared between the two taverns.
Ri=4,000 x €8 + 3,000 x €8 = €56,000 and Rj=3,000 x €10 = €30,000

Toi chooses P=€8 and Toj chooses P=€12⇒

Natives go to Tavern i because this is the cheapest. Tourists are shared between the two taverns.
Ri=4,000 x €8 + 3,000 x €8 = €56,000 and Rj=3,000 x €12 = €36,000

Toi chooses P=€10 and Toj chooses P=€10⇒

Natives are shared to the two taverns. Tourists are shared between the two taverns.
Ri=2,000 x €10 + 3,000 x €10 = €50,000 and Rj=2,000 x €10 + 3,000 x €10 = €50,000

Toi chooses P=€10 and Toj chooses P=€12⇒

Natives go to Tavern i because this is the cheapest. Tourists are shared between the two taverns.
Ri=4,000 x €10 + 3,000 x €10 = €70,000 and Rj=3,000 x €12 = €36,000

Toi chooses P=€12 and Toj chooses P=€12⇒

Natives are shared to the two taverns. Tourists are shared between the two taverns.
Ri=2,000 x €12 + 3,000 x €12 = €60,000 and Rj=2,000 x €12 + 3,000 x €12 = €60,000

Matrix form of the game

Payoffs: Total Revenues in € thousands		Tavern 2		
		Low (€8)	Medium (€10)	High (€12)
Tavern 1	Low (€8)	40, 40	56, 30	56, 36
	Medium (€10)	30, 56	50, 50	70, 36
	High (€12)	36, 56	36, 70	60, 60

The Nash equilibrium of the game:

If To1 chooses P=€8 the best response of To2 is P=€8
If To1 chooses P=€10 the best response of To2 is P=€8
If To1 chooses P=€12 the best response of To2 is P=€10
If To2 chooses P=€8 the best response of To1 is P=€8
If To2 chooses P=€10 the best response of To1 is P=€8
If To2 chooses P=€12 the best response of To1 is P=€10

The Nash equilibrium of the game is that both taverns' owners choose P=€8 and each make profits 40. Strong price competition leads firms to low profits.

2. Since the law does not allow taverns to price a daily menu below €9, in the new game, each tavern's owner must choose one out of two prices (Medium: €10; High: €12).

Hence, the matrix form of the game when tavern owners do not use the new pricing policy is:

Payoffs: Total Revenues in € thousands		Tavern 2	
		Medium (€10)	High (€12)
Tavern 1	Medium (€10)	50, 50	70, 36
	High (€12)	36, 70	60, 60

The Nash equilibrium of the game:
If To1 chooses P=€10 the best response of To2 is P=€10
If To1 chooses P=€12 the best response of To2 is P=€10
If To2 chooses P=€10 the best response of To1 is P=€10
If To2 chooses P=€12 the best response of To1 is P=€10
The Nash equilibrium of the game is that both taverns' owners choose P=€10 and each makes profits 50.

In this game, if the tavern owners use the new pricing policy, i.e., "*matching the rival's price if it is lower than mine*" we have:

- If To1 were to set P=€10 and To2 were to set P=€12, To2 will reduce its price to P=€10. R1=2,000 x €10 + 3,000 x €10 = €50,000 and R2=2,000 x €10 + 3,000 x €10 = €50,000
- If To2 were to set P=€10 and To1 were to set P=€12, To1 will reduce its price to P=€10 R1=2,000 x €10 + 3,000 x €10 = €50,000 and R2=2,000 x €10 + 3,000 x €10 = €50,000
- If To1 were to set P=€10 and To2 were to set P=€10, neither tavern changes its price and each tavern's revenues are €50,000.
- If To1 were to set P=€12 and To2 were to set P=€12, neither tavern changes its price and each tavern's revenues are €60,000.

Hence, the matrix form of the new game becomes:

Payoffs: Total Revenues in € thousands		Tavern 2	
		Medium (€10)	High (€12)
Tavern 1	Medium (€10)	50, 50	50, 50
	High (€12)	50, 50	60, 60

Two Nash equilibria: (P1: €10, P2: €10) and (P1: €12, P2: €12)
Focal point arguments point out to the (P1: €12, P2: €12) Nash equilibrium. That is, the game has two Pareto ranked equilibria, players will (most probably) coordinate on the Pareto superior one.

3. The pricing policy "matching the rival's price if it is lower than mine" increases both taverns' profits. The two tavern owners can coordinate on the equilibrium in which both are better off. Hence, the commitment to match the rival's price that seems to generate more competition, in the end acts as a mechanism that allows both tavern owners to increase their profits.