



**ΜΕΤΑΠΤΥΧΙΑΚΟ ΠΡΟΓΡΑΜΜΑ  
ΣΤΗ ΔΙΟΙΚΗΣΗ ΕΠΙΧΕΙΡΗΣΕΩΝ–MANAGEMENT ΤΟΥΡΙΣΜΟΥ  
(TOURISM MANAGEMENT)**

**Μάθημα – Οικονομική για Διοίκηση**

**Διδάσκων: Μιχάλης Πολέμης**

# Pricing of Multiple Products

- Products with Interrelated Demands
- Plant Capacity Utilization and Optimal Product Pricing
- Optimal Pricing of Joint Products
  - Fixed Proportions
  - Variable Proportions

# Pricing of Multiple Products

## Products with Interrelated Demands

For a two-product (A and B) firm, the marginal revenue functions of the firm are:

$$MR_A = \frac{\Delta TR_A}{\Delta Q_A} + \frac{\Delta TR_B}{\Delta Q_A}$$

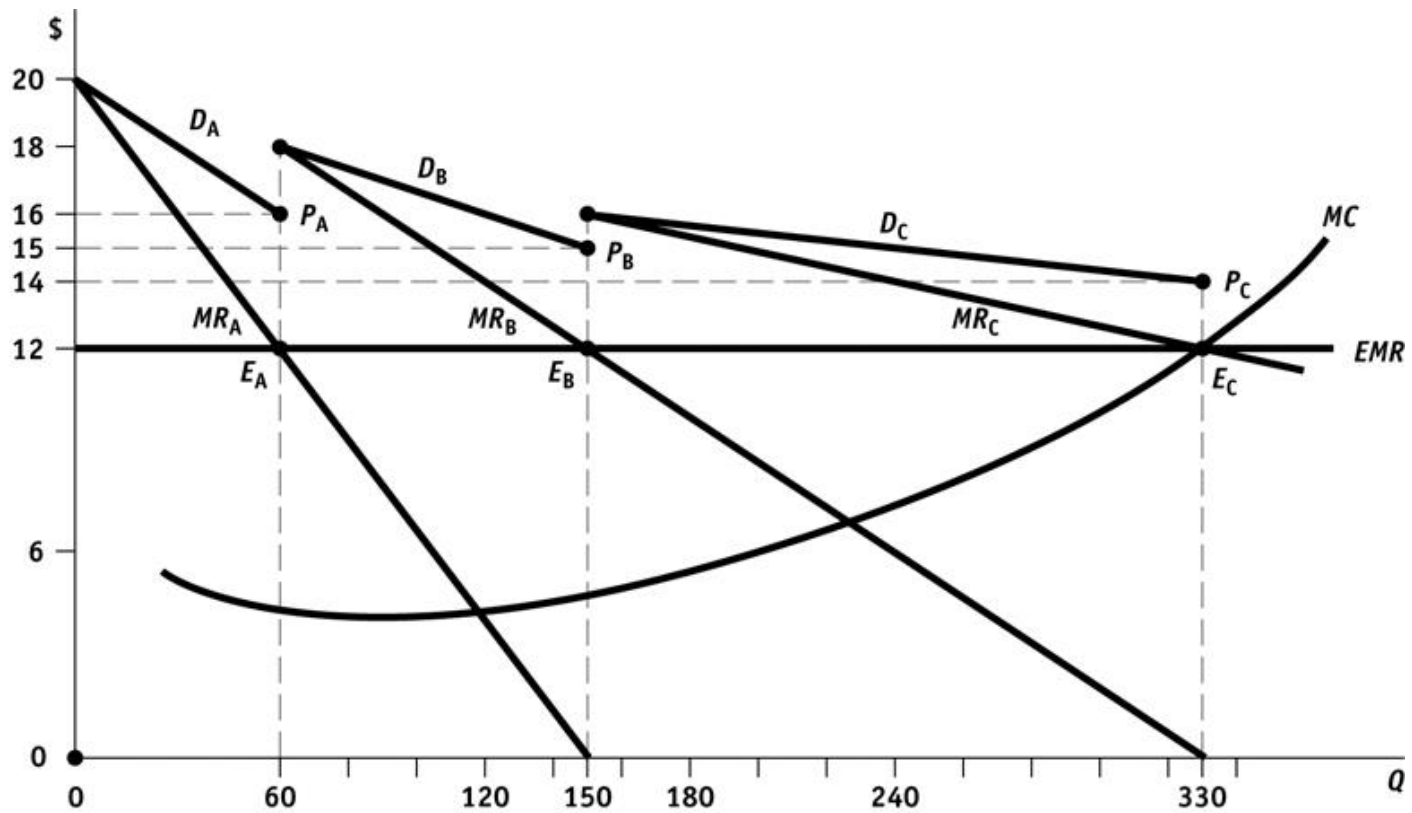
$$MR_B = \frac{\Delta TR_B}{\Delta Q_B} + \frac{\Delta TR_A}{\Delta Q_B}$$

# Pricing of Multiple Products

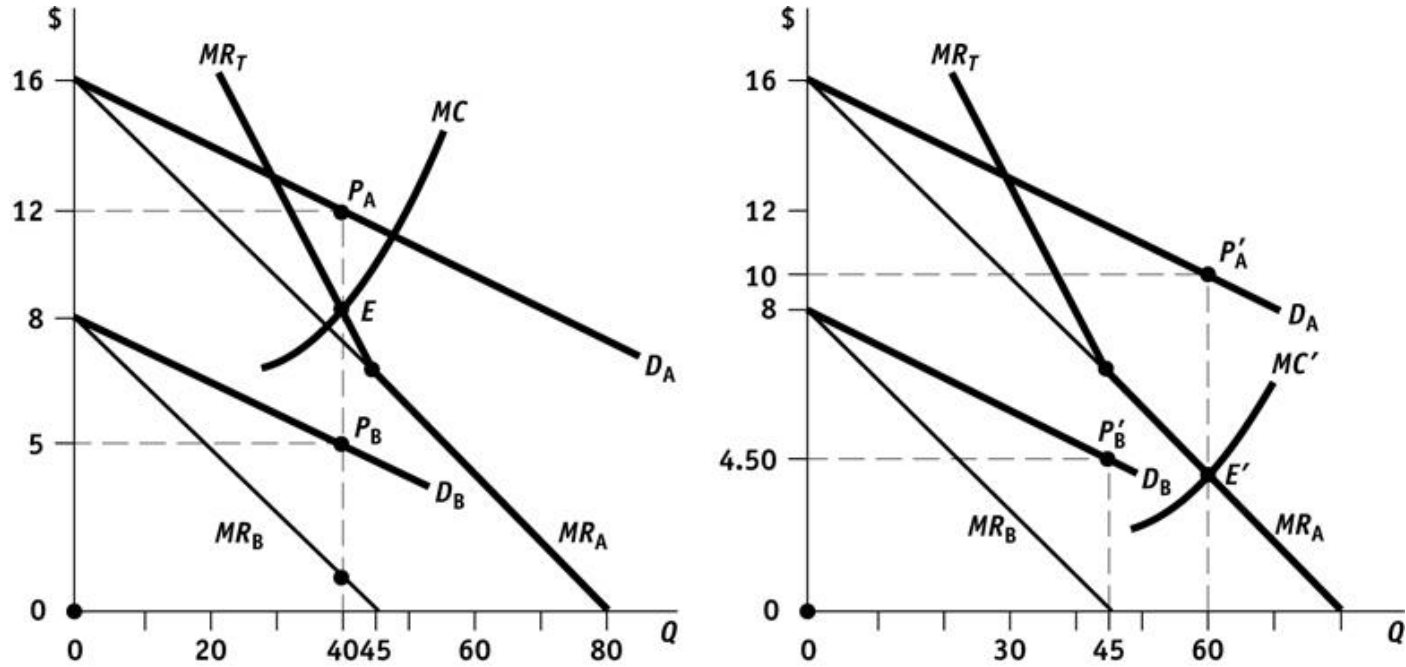
## Plant Capacity Utilization

A multi-product firm using a single plant should produce quantities where the marginal revenue ( $MR_i$ ) from each of its  $k$  products is equal to the marginal cost ( $MC$ ) of production.

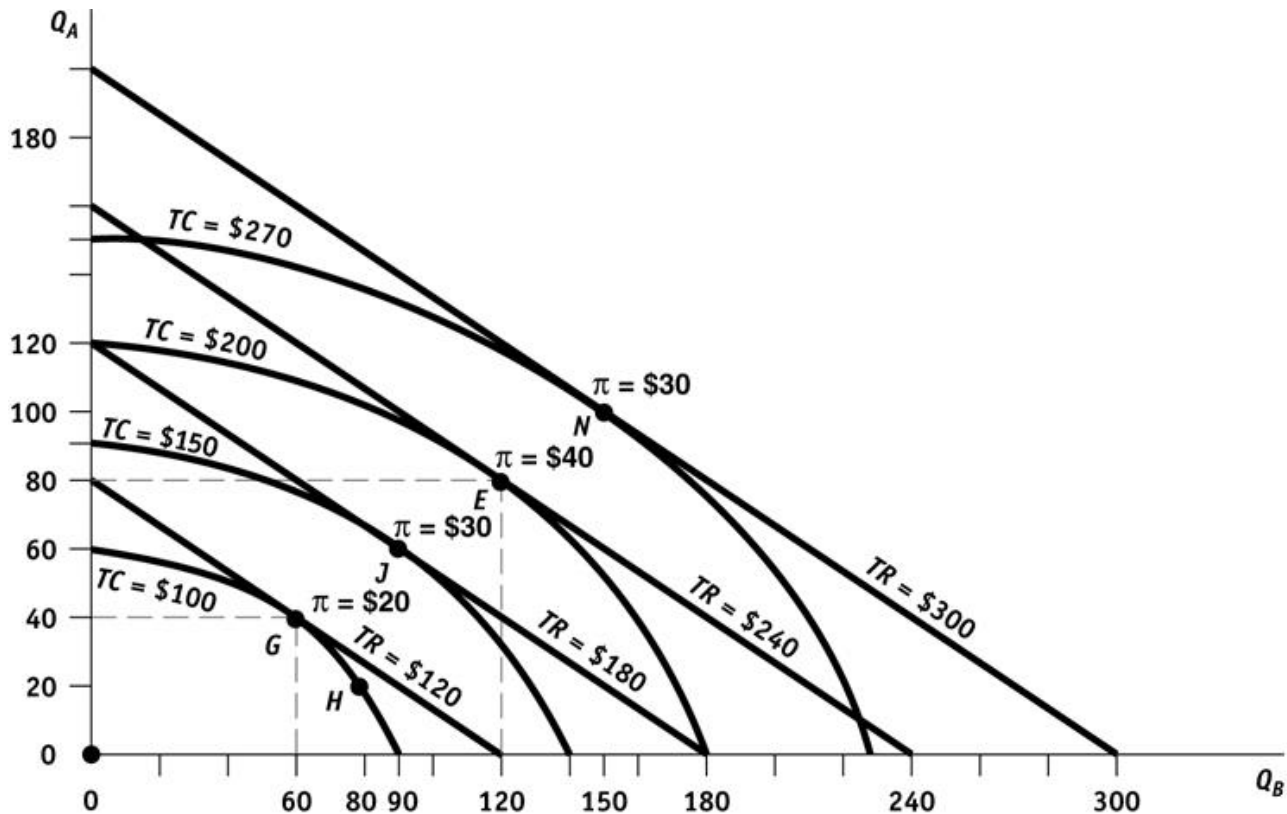
$$MR_1 = MR_2 = \dots = MR_k = MC$$



**FIGURE 11-1 Optimal Outputs and Prices of Multiple Products by a Firm**  $D_A$ ,  $D_B$ , and  $D_C$  are the demand curves for products A, B, and C sold by the firm, and  $MR_A$ ,  $MR_B$ , and  $MR_C$  are the corresponding marginal revenue curves. The firm maximizes profits when  $MR_A = MR_B = MR_C = MC$ . This is shown by points  $E_A$ ,  $E_B$ , and  $E_C$ , where the equal marginal revenue ( $EMR$ ) curve, at the level at which  $MR_C = MC$ , crosses the  $MR_A$ ,  $MR_B$ , and  $MR_C$  curves. Thus,  $Q_A = 60$  and  $P_A = \$16$ ;  $Q_B = 90$  (from  $150 - 60$ ) and  $P_B = \$15$ ; and  $Q_C = 180$  (from  $330 - 150$ ) and  $P_C = \$14$ . Note that each successive demand curve is more elastic and that the price of each successive product is lower, while its  $MC$  is higher.



**FIGURE 11-2 Optimal Output and Prices of Joint Products Produced in Fixed Proportions** In both panels,  $D_A$  and  $MR_A$ , and  $D_B$  and  $MR_B$  refer, respectively, to the demand and marginal revenue curves for products A and B, which are jointly produced in fixed proportions. The total marginal revenue ( $MR_T$ ) curve is obtained from the vertical summation of the  $MR_A$  and  $MR_B$  curves. When the marginal cost of the jointly produced production package is  $MC$  (see the left panel), the best level of output of products A and B is 40 units and is given by point  $E$ , at which  $MR_T = MC$ . At  $Q = 40$ ,  $P_A = \$12$  on  $D_A$  and  $P_B = \$5$  on  $D_B$ . On the other hand, with  $MC'$  (see the right panel), the best level of output of the joint product package is 60 units and is given by point  $E'$ , at which  $MR_T = MC'$ . At  $Q = 60$ ,  $P'_A = \$10$  on  $D_A$ , but since  $MR_B$  is negative for  $Q_B > 45$ , the firm sells only 45 units of product B at  $P'_B = \$4.50$  (at which  $TR_B$  is maximum at  $MR_B = 0$ ) and disposes of the remaining 15 units of product B.



**FIGURE 11-3 Profit Maximization with Joint Products Produced in Variable Proportions** The curved lines are product transformation curves showing the various combinations of products A and B that the firm can produce at each level of total cost ( $TC$ ). The curvature arises because the firm's productive resources are not perfectly adaptable in the production of products A and B that give rise to the same total revenue ( $TR$ ) to the firm when sold at constant prices. The tangency point of an isorevenue to a  $TC$  curve gives the combination of products A and B that leads to the maximum profit ( $\pi$ ) for the firm for the specific  $TC$ . The overall maximum profit of the firm is  $\pi = \$40$ . This is earned by producing and selling 80A and 120B (point E) with  $TR = \$240$  and  $TC = \$200$ .

# Price Discrimination

Charging different prices for a product when the price differences are not justified by cost differences.

Objective of the firm is to attain higher profits than would be available otherwise.



# Price Discrimination

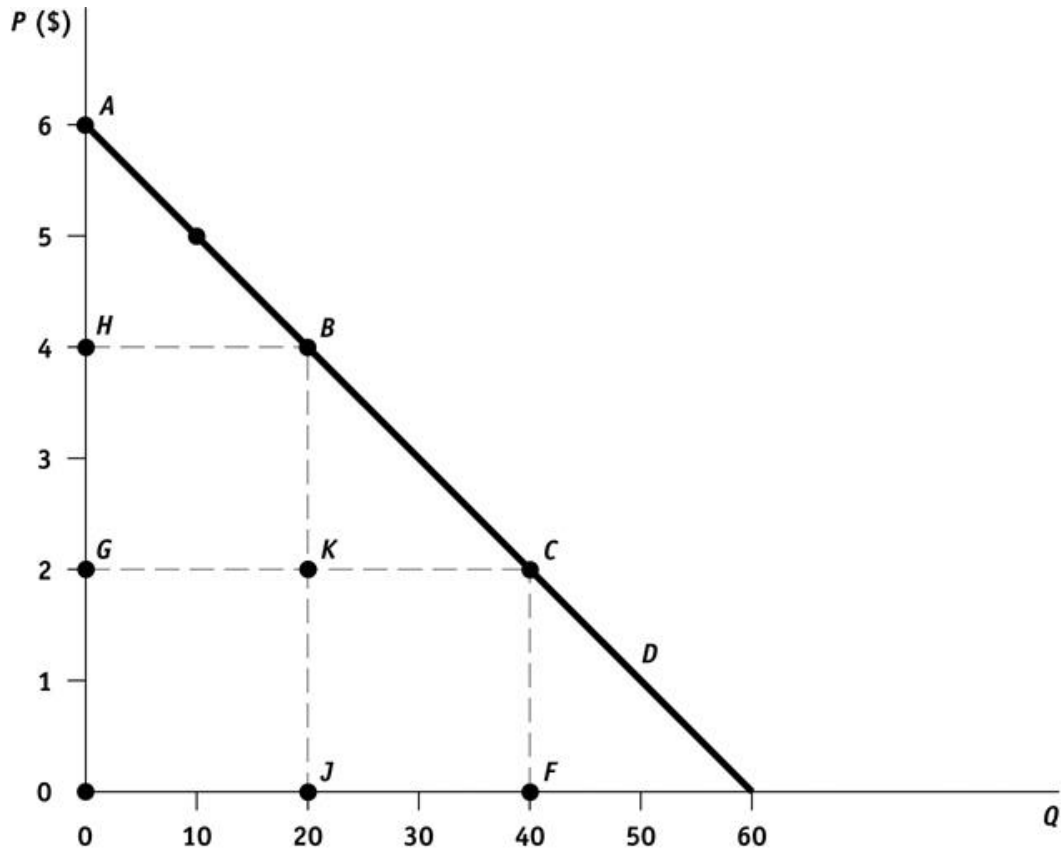
1. Firm must be an imperfect competitor (a price maker)
2. Price elasticity must differ for units of the product sold at different prices
3. Firm must be able to segment the market and prevent resale of units across market segments

# First-Degree Price Discrimination

- Each unit is sold at the highest possible price
- Firm extracts all of the consumers' surplus
- Firm maximizes total revenue and profit from any quantity sold

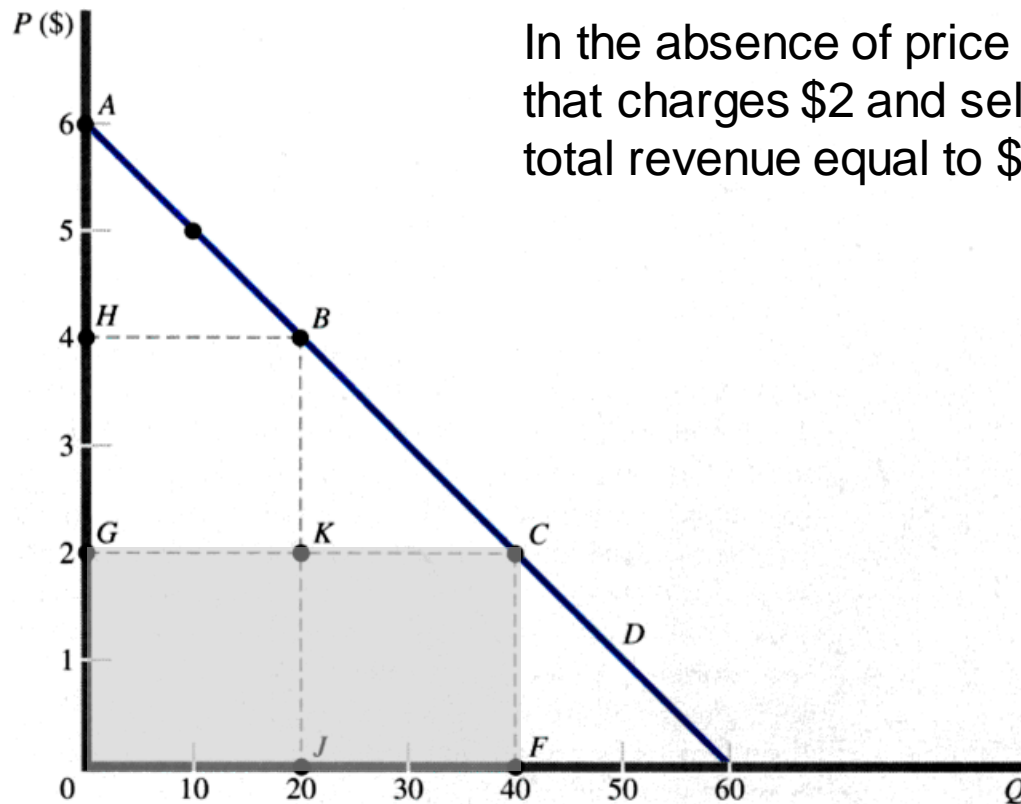
# Second-Degree Price Discrimination

- Charging a uniform price per unit for a specific quantity, a lower price per unit for an additional quantity, and so on
- Firm extracts part, but not all, of the consumers' surplus



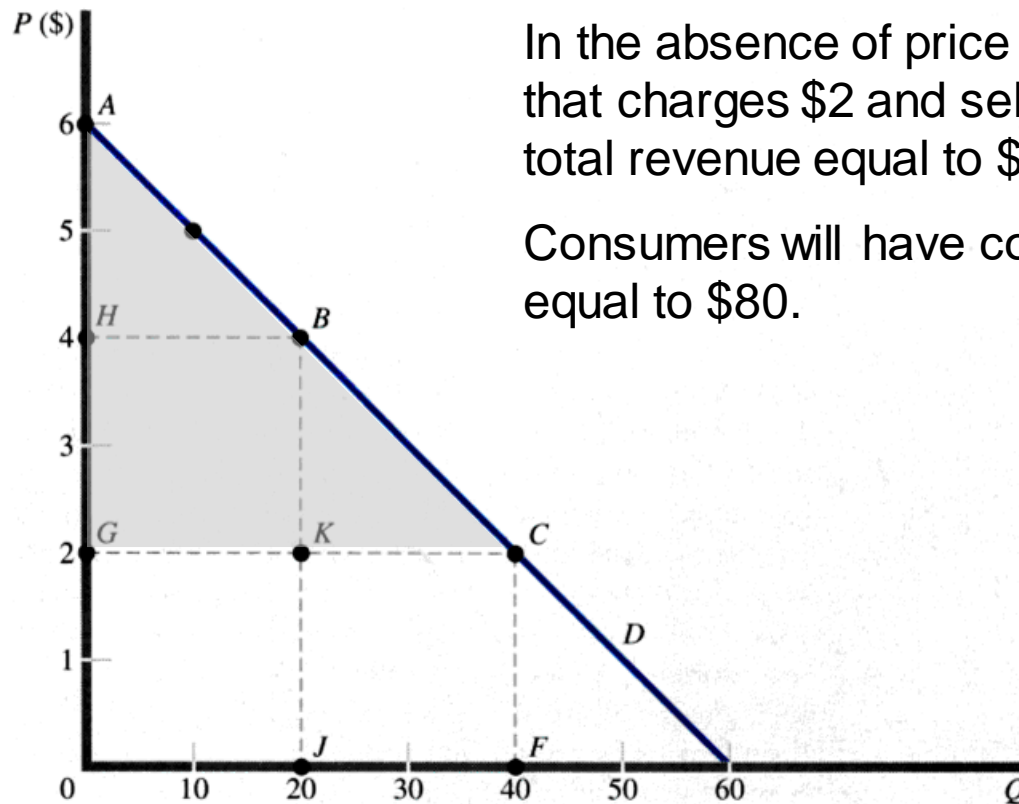
**FIGURE 11-4 First- and Second-Degree Price Discrimination** With  $D$  as the demand curve faced by a monopolist, the firm could sell  $Q = 40$  at  $P = \$2$  for a  $TR = \$80$  (the area of rectangle  $CF0G$ ). Consumers, however, would be willing to pay  $ACF0 = \$160$  for 40 units of the product. The difference of  $\$80$  (the area of triangle  $ACG$ ) is the consumer's surplus. With first-degree price discrimination (i.e., by selling each unit of the product separately at the highest price possible), the firm can extract all the consumers' surplus from consumers. If, however, the firm charged the price of  $P = \$4$  per unit for the first 20 units of the product and  $\$2$  per unit on the next 20 units, the total revenue of the firm would be  $\$120$  (the sum of the areas of rectangles  $BJ0H$  and  $CFJK$ ), so that the firm would extract  $\$40$  (the area of rectangle  $BKGH$ ), or half of the consumers' surplus from consumers. This is second-degree price discrimination.

# First- and Second-Degree Price Discrimination



In the absence of price discrimination, a firm that charges \$2 and sells 40 units will have total revenue equal to \$80.

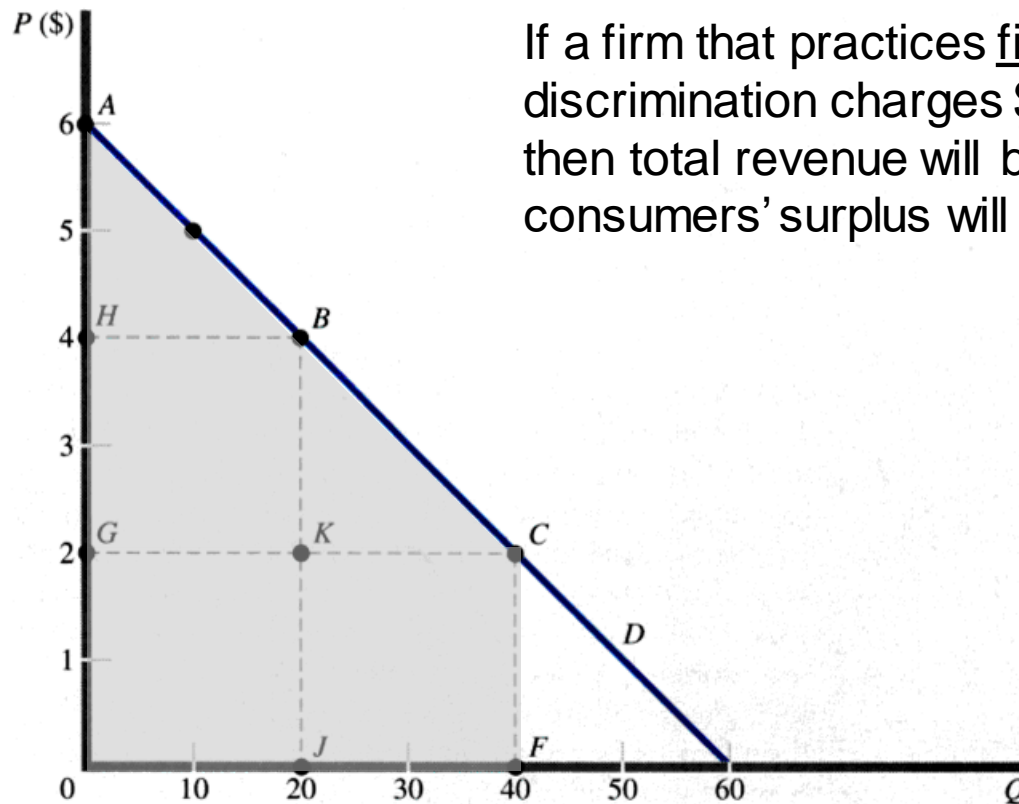
# First- and Second-Degree Price Discrimination



In the absence of price discrimination, a firm that charges \$2 and sells 40 units will have total revenue equal to \$80.

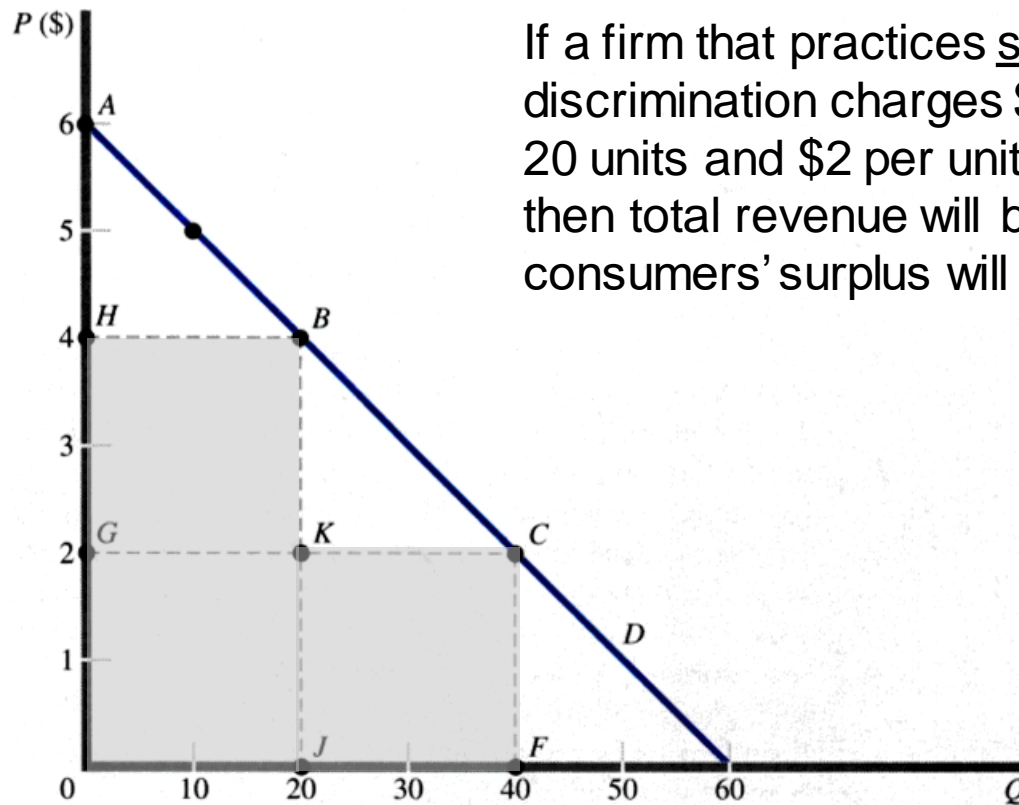
Consumers will have consumers' surplus equal to \$80.

# First- and Second-Degree Price Discrimination



If a firm that practices first-degree price discrimination charges \$2 and sells 40 units, then total revenue will be equal to \$160 and consumers' surplus will be zero.

# First- and Second-Degree Price Discrimination



If a firm that practices second-degree price discrimination charges \$4 per unit for the first 20 units and \$2 per unit for the next 20 units, then total revenue will be equal to \$120 and consumers' surplus will be \$40.



# Third-Degree Price Discrimination

- Charging different prices for the same product sold in different markets
- Firm maximizes profits by selling a quantity on each market such that the marginal revenue on each market is equal to the marginal cost of production

# Third-Degree Price Discrimination

$$Q_1 = 120 - 10 P_1 \text{ or } P_1 = 12 - 0.1 Q_1 \text{ and } MR_1 = 12 - 0.2 Q_1$$

$$Q_2 = 120 - 20 P_2 \text{ or } P_2 = 6 - 0.05 Q_2 \text{ and } MR_2 = 6 - 0.1 Q_2$$

$$MR_1 = MC = 2$$

$$MR_2 = MC = 2$$

$$MR_1 = 12 - 0.2 Q_1 = 2$$

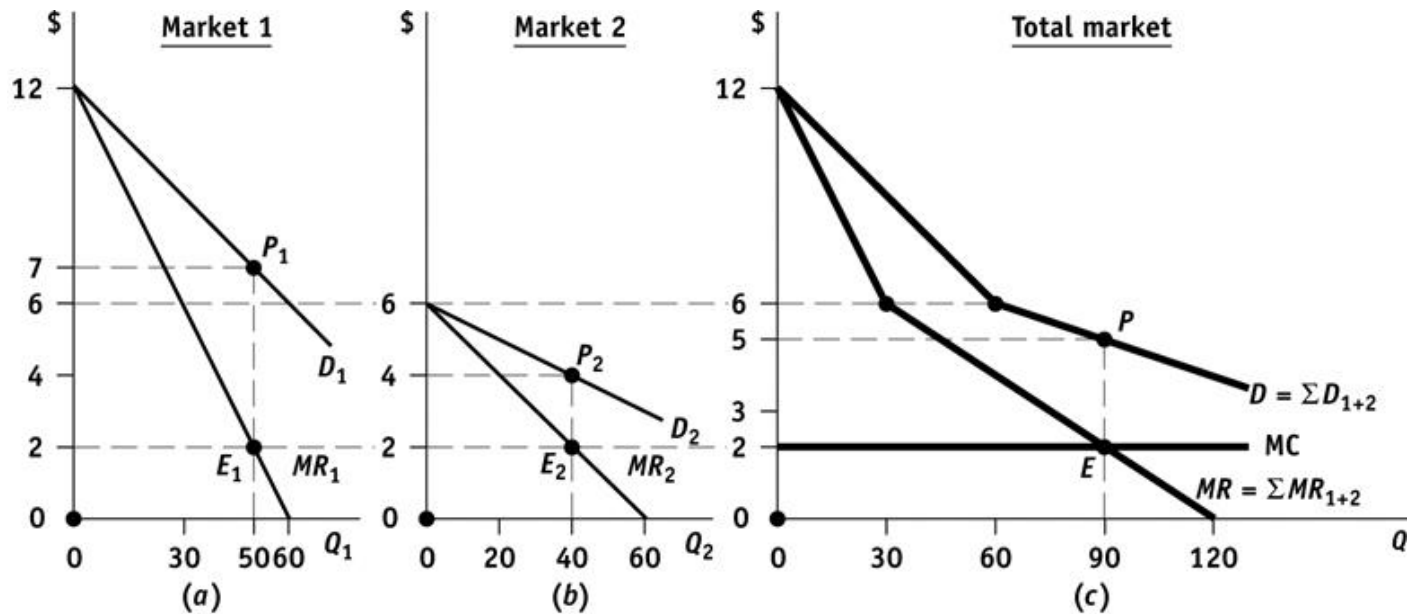
$$MR_2 = 6 - 0.1 Q_2 = 2$$

$$Q_1 = 50$$

$$Q_2 = 40$$

$$P_1 = 12 - 0.1 (50) = \$7$$

$$P_2 = 6 - 0.05 (40) = \$4$$



**FIGURE 11-5 Third-Degree Price Discrimination** Panel *a* shows  $D_1$  and  $MR_1$  (the demand and marginal revenue curves faced by the firm in market 1), panel *b* shows  $D_2$  and  $MR_2$ , and panel *c* shows  $D$  and  $MR$  (the total demand and marginal revenue curves for the two markets together).  $D = \sum D_{1+2}$ , and  $MR = \sum MR_{1+2}$ , by horizontal summation. The best level of output of the firm is 90 units and is given by point  $E$  in panel *c* at which  $MR = MC = \$2$ . The firm sells 50 units of the product in market 1 and 40 units in market 2, so that  $MR_1 = MR_2 = MR = MC = \$2$  (see points  $E_1$ ,  $E_2$ , and  $E$ ). For  $Q = 50$ ,  $P_1 = \$7$  on  $D_1$  in market 1, and for  $Q_2 = 40$ ,  $P_2 = \$4$  on  $D_2$  in market 2. With an average total costs of \$3 per unit for  $Q = 90$ , the firm earns a profit of \$4 per unit and \$200 in total in market 1, and \$1 per unit and \$40 in total in market 2, for an overall total profit of \$240 in both markets. In the absence of price discrimination,  $Q = 90$ ,  $P = \$5$  (see panel *c*), so that profits are \$2 per unit and \$180 in total.

**TABLE 11-1** Electricity Rates Charged by Con Edison in 2005 (cents per kilowatt-hour)

	kWh	Cents/kWh	kWh	Cents/kWh
Residential Rates (Single Residence)				
June–September	0–250	5.18	Above 250	5.83
Other months	0–250	5.18	Above 250	4.76
Commercial Rates (Small Business)				
June–September	0–900	6.97	Above 900	6.23
April	0–900	5.96	Above 900	5.23
Commercial Rates (Large Business)				
Low tension	0–15,000	1.42	Above 15,000	1.42
High tension	0–15,000	1.33	Above 15,000	1.32

*Source:* Con Edison, New York City (2005).

**TABLE 11-2 Electricity Rates Charged by Con Edison in 2005 for Peak and Off-Peak Hours (cents per kilowatt-hour)**

	Peak Hours	Off-Peak Hours
Residential Rates, 0–250 kWh (single residence)		
June–September	18.26	0.63
Other months	6.57	0.63
Commercial Rates, 0–900 kWh (small business)		
June–September	13.55	0.48
Other months	6.66	0.48
Delivery Charges (Plus \$/kWh) for Low and High Tension (large business)		
June–September	10.24 (0.52)	5.47 (0.52)
Other months	7.55 (0.52)	3.27 (3.17)

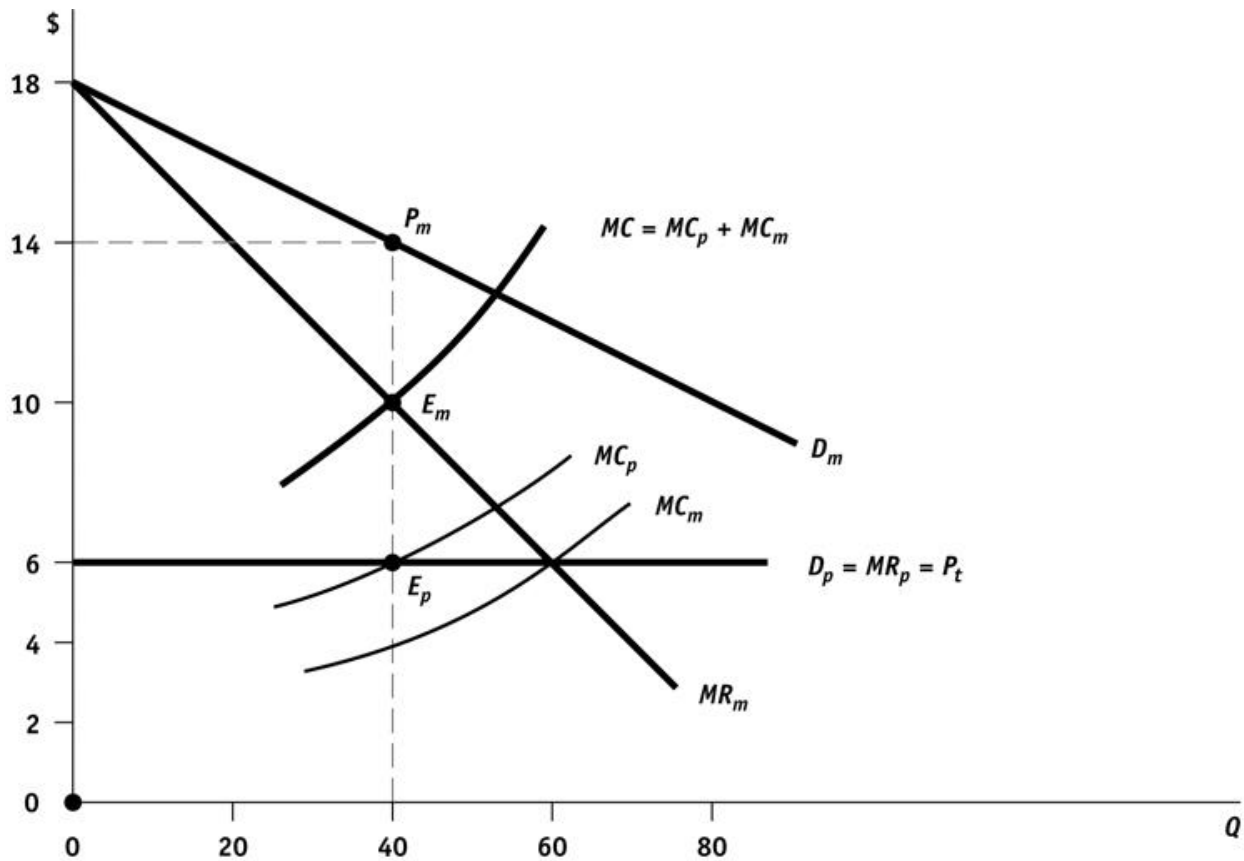
*Source:* Con Edison, New York City (2005).

# International Price Discrimination

- Persistent Dumping
- Predatory Dumping
  - Temporary sale at or below cost
  - Designed to bankrupt competitors
  - Trade restrictions apply
- Sporadic Dumping
  - Occasional sale of surplus output

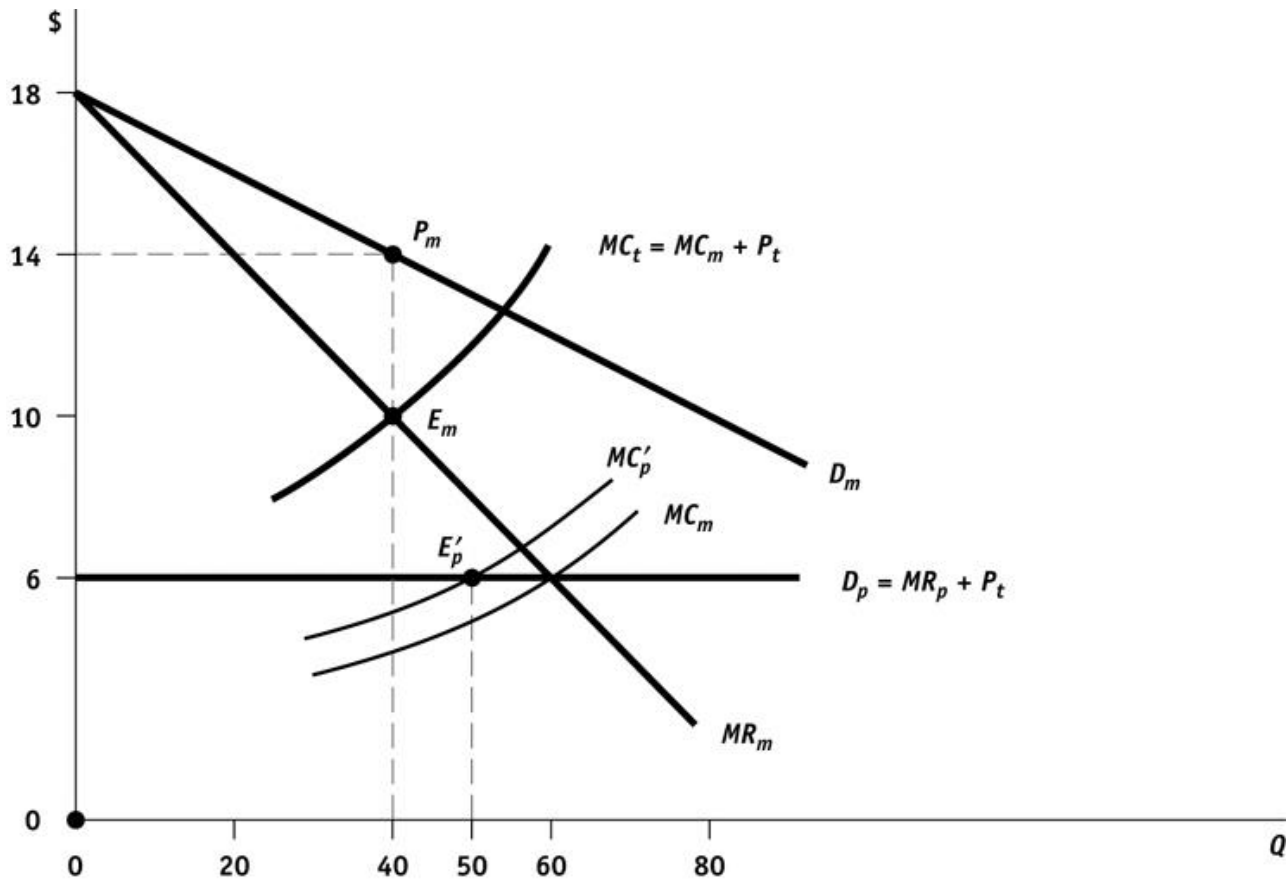
# Transfer Pricing

- Pricing of intermediate products sold by one division of a firm and purchased by another division of the same firm
- Made necessary by decentralization and the creation of semiautonomous profit centers within firms



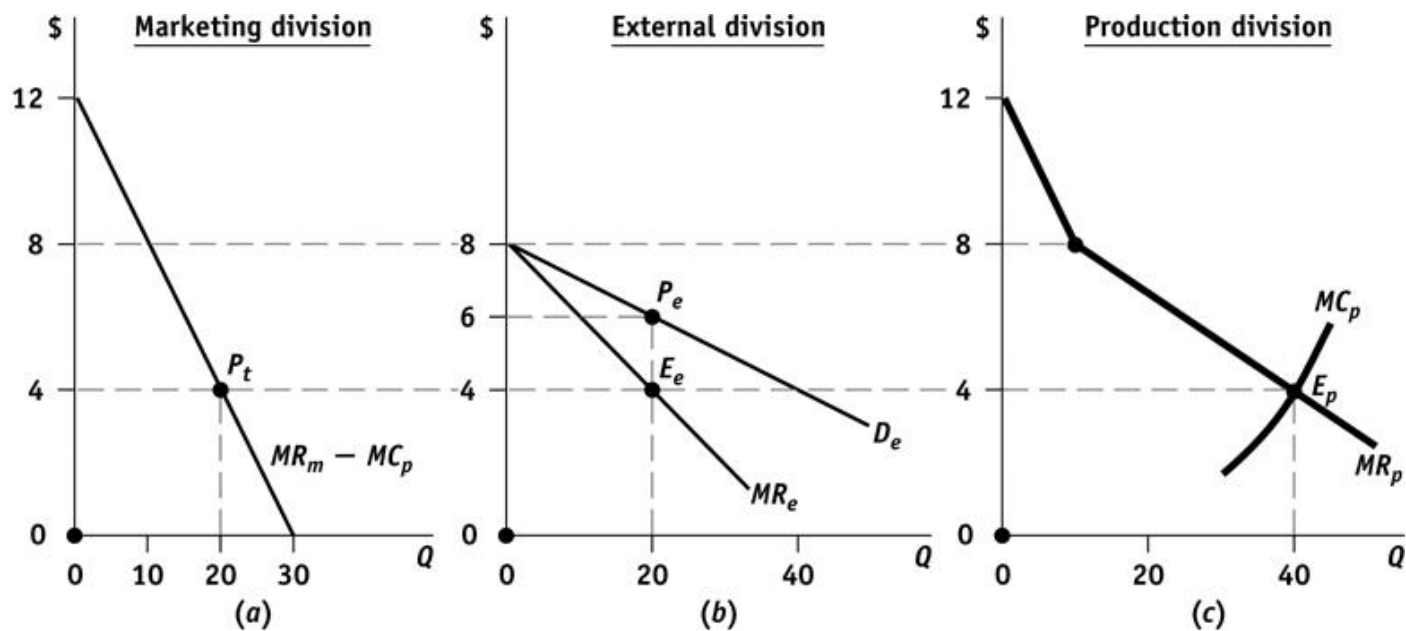
**FIGURE 11-6 Transfer Pricing of the Intermediate Product with No External Market**  $MC$ , the marginal cost of the firm, is equal to the vertical summation of  $MC_p$  and  $MC_m$ , the marginal cost curves of the production and the marketing divisions of the firm, respectively.  $D_m$  is the external demand for the final product faced by the marketing division of the firm, and  $MR_m$  is the corresponding marginal revenue curve. The firm's best level of output of the final product is 40 units and is given by point  $E_m$ , at which  $MR_m = MC$ , so that  $P_m = \$14$ . Since the production of each unit of the final product requires 1 unit of the intermediate product, the transfer price for the intermediate product,  $P_t$ , is set equal to  $MC_p$  at  $Q_p = 40$ . Thus,  $P_t = \$6$ . With  $D_p = MR_p = P_t = MC_p = \$6$  at  $Q_p = 40$  (see point  $E_p$ ),  $Q_p = 40$  is the best level of output of the intermediate product for the production division.





**FIGURE 11-7 Transfer Pricing of the Intermediate Product with a Perfectly Competitive Market**

This figure is identical to Figure 11-6, except that  $MC$  is lower than  $MC_p$ . At the perfectly competitive external price of  $P_t = \$6$  for the intermediate product, the production division of the firm faces  $D_p = MR_p = P_t = \$6$ . Therefore, the best level of output of the intermediate product is  $Q_p = 50$  and is given by point  $E'_p$  at which  $D_p = MR_p = P_t = MC = \$6$ . Since the marketing division can purchase the intermediate product (internally or externally) at  $P_t = \$6$ , its total marginal cost curve,  $MC_t$ , is equal to the vertical summation of  $MC_m$  and  $P_t$ . Thus, the best level of output of the final product by the marketing division is 40 units and is given by point  $E_m$ , at which  $MR_m = MC_t$ , so that  $P_m = \$14$  (as in Figure 11-6).



**FIGURE 11-8 Transfer Pricing of the Intermediate Product with an Imperfectly Competitive Market** Panel *a* presents the net marginal revenue ( $MR_m - MC_p$ ) curve of the marketing division for the intermediate product, panel *b* shows the external demand and marginal revenue curves (that is,  $D_e$  and  $MR_e$ ) for the intermediate product, while panel *c* shows the  $MR_p = MR_m - MC_p + MR_e$  and  $MC_p$  curves of the production division. The best level of output of the intermediate product by the production division is 40 units and is given by  $E_p$ , at which  $MR_p = MC_p$  in panel *c*. The optimal distribution of  $Q_p = 40$  is 20 units to the marketing division and 20 units to the external market (given by points  $P_t$  and  $E_e$ , respectively), at which  $MR_m - MC_p = MR_e = MR_p = MC_p = \$4$ . The transfer price to the marketing division is then  $P_t = MC_p = \$4$ , and the price of the intermediate product for sales on the external market is  $P_e = \$6$ .

# Pricing in Practice

## Cost-Plus Pricing

- Markup or Full-Cost Pricing
- Fully Allocated Average Cost (C)
  - Average variable cost at normal output
  - Allocated overhead
- Markup on Cost ( $m$ ) =  $(P - C)/C$
- Price =  $P = C (1 + m)$

# Pricing in Practice

## Optimal Markup

$$P = MR \left( \frac{E_P}{E_p + 1} \right)$$

$$MR = C$$

$$P = C \left( \frac{E_P}{E_p + 1} \right)$$

# Pricing in Practice

## Optimal Markup

$$P = C \left( \frac{E_P}{E_p + 1} \right)$$

$$P = C(1 + m)$$

$$C(1 + m) = C \left( \frac{E_P}{E_p + 1} \right)$$

$$m = \frac{E_P}{E_p + 1} - 1$$

# Pricing in Practice

## Incremental Analysis

A firm should take an action if the incremental increase in revenue from the action exceeds the incremental increase in cost from the action.

# Pricing in Practice

- Two-Part Tariff
- Tying
- Bundling
- Prestige Pricing
- Price Lining
- Skimming
- Value Pricing

## Example: Price Discrimination

*Red Rose* is a monopolist selling flowers in two villages, namely, *Trenton* and *Stenton*. There are 100 individuals in *Trenton*, each with a demand function for flowers  $q_T = 0.4 - \frac{p_T}{100}$ . There are 500 individuals in *Stenton*, each with a demand function for flowers  $q_S = 0.16 - \frac{p_S}{500}$ . *Red Rose's* cost function is  $C(Q) = 10Q$ , where  $Q = Q_T + Q_S$  is the total volume of flowers sold.

1. Suppose that price discrimination is possible. Find the equilibrium and the price elasticities of demand in each village. Comment on your findings.

**Answer:** The aggregate demand function for *Trenton* is  $Q_T = q_T \times 100 = 40 - p_T$  whereas that for *Stenton* is  $Q_S = q_S \times 500 = 80 - p_S$ . Accordingly, the total revenue functions are:  $TR_T = p_T(Q_T) \times Q_T = 40Q_T - Q_T^2$  and  $TR_S = p_S(Q_S) \times Q_S = 80Q_S - Q_S^2$ . This implies that  $MR_T = 40 - 2Q_T$  and  $MR_S = 80 - 2Q_S$ . In equilibrium, it holds  $MR_T = MR_S = MC (= 10)$ .

Substituting from above and solving the resulting equations one gets  $Q_T = 15$ ,  $Q_S = 35$ ,  $p_T = 25$  and  $p_S = 45$ . The price elasticities of demand are  $\varepsilon_T^d = \frac{\partial Q_T}{\partial p_T} \frac{p_T}{Q_T} = -1 \times \frac{25}{15} = -1.67$  and  $\varepsilon_S^d = \frac{\partial Q_S}{\partial p_S} \frac{p_S}{Q_S} = -1 \times \frac{45}{35} = -1.29$ .

The results make economic sense; the higher price is charged in the market where the elasticity of demand is lower.



## Example: Price Discrimination

2. Suppose that price discrimination is not possible. Find the new equilibrium.

**Answer:** The aggregate (two-markets) demand is  $Q = Q_T + Q_S = 120 - 2p$ , where  $p$  is the common price. The respective marginal revenue function is  $MR = 60 - Q$ , which when equated to the marginal cost  $MC = 10$  yielding  $Q = 50$  and  $p = 35$ .

3. Compare *Red Rose's* profits, aggregate (village) consumer surpluses and total welfare under both scenarios.

**Answer:** With the discriminatory price policy the firm's profit is  $\pi = (15 \times 25) + (35 \times 45) - (10 \times 50) = 1,450$ . Consumer surpluses are:  $CS_T = 0.5 \times (40 - 25) \times 15 = 112.5$  and  $CS_S = 0.5 \times (80 - 45) \times 35 = 612.5$ . Therefore, the two-market consumer surplus is  $CS = 725$  and the welfare level is  $W = 1,450 + 725 = 2,175$ .

With the non-discriminatory pricing the firm's profit is  $\pi = (50 \times 35) - (10 \times 50) = 1,250$ . The consumer surplus in *Trenton* is  $CS_T = 0.5 \times (40 - 35) \times 5 = 12.5$  and in *Stenton* is  $CS_S = 0.5 \times (80 - 35) \times 45 = 1,012.5$ . The corresponding welfare level is  $W = 1,250 + 12.5 + 1,012.5 = 2,275$  (increases by 100).