

# Lectures on Economic Growth

## Topic: Productivity

an upper intermediate course offered at the 7th semester at the

Economics Department, University of Piraeus



## Previously in this course:

- We have added several factors to the Solow model:
  - Physical capital
  - Raw labor
  - Human capital
- Today we go from

$$Y = f(K, h \cdot L)$$

to

$$Y = A \cdot f(K, h \cdot L)$$

- By adding productivity  $A$ , we open yet another way for countries to have different GDP per capita. We also start working on explaining  $A$ .

## Previously in this course:

- By adding productivity  $A$ , we open yet another way for countries to have different GDP per capita. We also start working on explaining  $A$ .
- Finally, you might say. Factors are interesting, but technology must also be a very important determinant of growth. Growth in living standards is so wrapped up with technological progress that often the two are indistinguishable.
- However, analysis of technology is much more slippery than that of factors.

# Today

Chapters 7 and 8 of Weil, which are about productivity and the role of technology.

- Is productivity the same as technology?
- Where does technological progress come from?
- What do the economic dynamics look like?
- We modify the Solow model one last time, to account for continuous changes in  $A$ .

## Part I: Productivity

We have looked at factor accumulation to explain differences in production. Countries with the same stock of factors can still have different levels of GDP per capita: they have different productivity.

$$Y = A \cdot K^\alpha \cdot (h \cdot L)^{1-\alpha}$$

Divide by L to get per capita values

$$y = A \cdot (k^\alpha h^{1-\alpha})$$

This splits production in productivity and factors.

## Comparing countries

$$y = A \cdot (k^\alpha h^{1-\alpha})$$

means that the ratio of two GDP per capita is

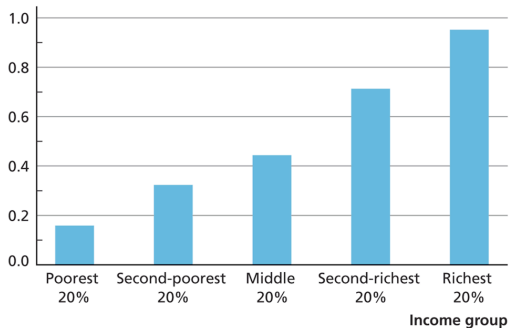
$$\frac{y_1}{y_2} = \frac{A_1}{A_2} \cdot \frac{k_1^\alpha h_1^{1-\alpha}}{k_2^\alpha h_2^{1-\alpha}}$$

There is data on

- $y$ : GDP/head (not per hour worked, unfortunately).
- $k$ : Estimate  $K$  by the perpetual inventory method.
- $h$ : Use educational attainment and data on the return to schooling.
- $\alpha$ : Take the conventional value of  $1/3$ .

Use these data to compute  $A_1/A_2$ : development accounting. Since it's all relative, use a numéraire: USA.

# Trends in factors

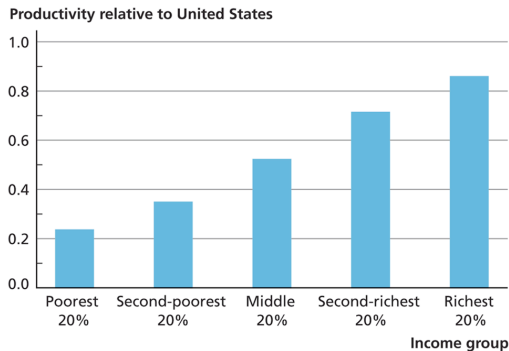
**FIGURE 7.3****Role of Factors of Production in Determining Output per Worker, 1998****Factors of production per worker relative to United States**

For sources, see Table 7.2.

Note that for the poorest countries, the gap with the US is mostly

# Trends in productivity

**FIGURE 7.4**  
**Role of Productivity in Determining Output per Worker, 1998**



For sources, see Table 7.2.

Note that for the richest countries, the gap with the US is mostly productivity.

# Growth decomposition

Production comes from multiple factors. A change in production (growth) can be explained by changes in these factors. Let's take a slightly more general function, and leave out human capital:

$$Y_t = F(A_t, K_t, L_t)$$

Changes in  $Y_t$  are the result of changes in  $L_t$ ,  $K_t$ ,  $A_t$ . But how large is the contribution of each?

## Growth decomposition 2

How do different factors affect production? That depends on  $F(\cdot)$ .

Define

$$F_A = \frac{dF}{dA} \quad F_L = \frac{dF}{dL} \quad F_K = \frac{dF}{dK}$$

Then

$$\frac{dY_t}{dt} = F_A \cdot \frac{dA_t}{dt} + F_K \cdot \frac{dK_t}{dt} + F_L \cdot \frac{dL_t}{dt}$$

## Usual suspects

Let's look at these terms:

$\frac{dY_t}{dt}$	$(\dot{Y})$	known	to be explained
$\frac{dL_t}{dt}$	$(\dot{L})$	known	from authorities
$\frac{dK_t}{dt}$	$(\dot{K})$	known	from investments, writeoffs
$\frac{dA_t}{dt}$	$(\dot{A})$	unknown	measure of our ignorance

So if we know  $F_A$ ,  $F_L$  and  $F_K$ , we can tie down technological progress.

## Factor shares

But we do know something about  $F_L$  and  $F_K$ : factors are supposed to be paid their marginal product.

- $dY/dK = dF/dK = r$  (with  $r$  the gross rate of interest [which is the nominal rate plus  $\delta$ , writeoffs])
- $dY/dL = dF/dL = w$  (with  $w$  the wage rate).

Now if we assume that productivity is multiplicative,

$$Y_t = F(A_t, K_t, L_t) = A_t \cdot G(K_t, L_t)$$

then

$$F_A = G(K_t, L_t) = Y_t/A_t$$

(remember this for a second)

# Rewrite

Now for some mathematical gymnastics:

$$\frac{dY_t}{dt} = F_A \cdot \frac{dA_t}{dt} + F_K \cdot \frac{dK_t}{dt} + F_L \cdot \frac{dL_t}{dt}$$

$$\dot{Y} = F_A \cdot \dot{A} + r \cdot \dot{K} + w \cdot \dot{L}$$

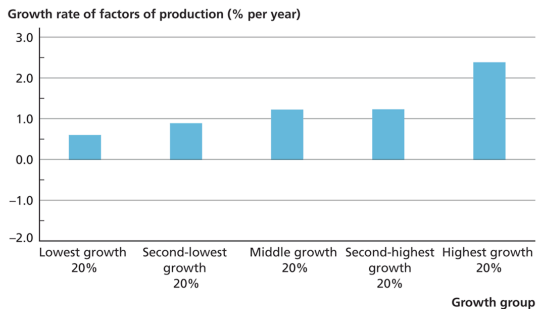
$$\frac{\dot{Y}}{Y} = F_A \cdot \frac{\dot{A}}{Y} + r \cdot \frac{\dot{K}}{Y} + w \cdot \frac{\dot{L}}{Y}$$

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{r \cdot K}{Y} \cdot \frac{\dot{K}}{K} + \frac{w \cdot L}{Y} \cdot \frac{\dot{L}}{L}$$

$$g_Y = s_R + s_K \cdot g_K + s_L \cdot g_L$$

with  $s_K$  the share of capital in gdp and  $s_L$  the share of labor in gdp.  
Sources of growth equation.

# Trends in factors

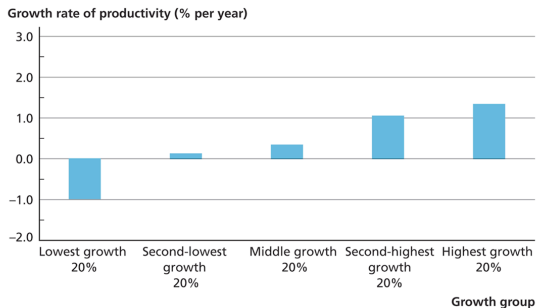
**FIGURE 7.5****Role of Factors of Production in Determining Growth, 1960–1998**

For sources, see Table 7.2.

Note that the bottom 80% grows almost equally fast.

# Trends in productivity

**FIGURE 7.6**  
Role of Productivity in Determining Growth, 1960–1998



For sources, see Table 7.2.

This appears to be the more important factor explaining growth differences.

# Conclusions

- Productivity differs among countries: poor countries produce only a quarter with the same factors.
- According to Weil, 57% of income differences is explained by factors, 43% by productivity.
- Productivity growth varies from  $-1.0\%$  to  $+1.3\%$  for quintiles.
- According to Weil, 42% of income growth differences are explained by changes in factors, 58% by changes in productivity.

## Part II: Technological progress

In our production function,

$$Y = A \cdot F(K, h \cdot L)$$

productivity tells you how much you can make with a set of factors. Progress is when  $A$  increases; this leads to economic growth. Where do increases in  $A$  come from?

According to Weil,  $A$  is a product of efficiency and technology. Changes in technology, these days, come mostly from formal R&D.

To understand R&D, we must understand a few things about the nature of technology.

## Rival, Nonrival

Most goods and services you can think of share two qualities: they are rival and excludable.

Rival: if someone consumes the good or service, somebody else cannot.

- Icecream, haircut, etc.

Nonrival: if someone consumes the good or service, it is still there for everybody else.

- Concert performance, flood protection, weather forecast, software, lectures.

The costs of providing the good to other people, once some people have it, are zero.

## Excludable, Nonexcludable

Excludable: It is possible to charge each buyer separately for the product.

- Car, donut, movie, ringtones.

Nonexcludable: When the good or service is provided to one person, everybody else gets it (or at least you cannot keep it from other people).

- Broadcast, flood protection, jokes.

## Why bother?

Normally, markets are pretty good at coordinating activities.

However, when goods are nonrival and nonexcludable, markets typically provide too little of these goods.

(Usually, the government steps in: defence, flood protection)

Ideas are nonrival and nonexcludable. And they drive technological progress.

This is good: everybody profits from a good idea.

This is bad: there is little incentive to produce them.

## Wright Brothers

Started a career repairing bicycles, but used the proceeds to experiment with flight.

Built a wind tunnel in their bike repair shop to experiment with wing design.

December 17, 1903: Orville flies 39 meters in 12 seconds; Wilbur flies 279 meters in 59 seconds.

These flights take place almost without public. The brothers have constructed their plane in secret and applied for a patent.



<http://www.wikipedia.org>

# Wright Brothers

No. 821,393

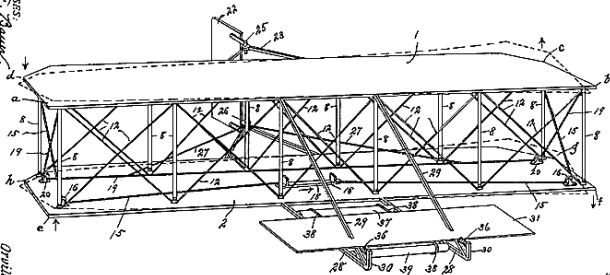
PATENTED MAY 22, 1906

O. & W. WRIGHT,  
FLYING MACHINE.

APPLICATION FILED MAR. 23, 1903

3 SHEETS—SHEET 1.

FIG. 1.



WITNESSES:  
William F. Bauer.

Ernie Miller.

INVENTORS:  
Orville Wright,  
Walter Wright.

BY  
H. A. GARDNER,  
ATTORNEY.

Source: <http://msstate.edu>

## Then what?

So do the Wright brothers become rich?

Well no: The Wright brothers' success as plane builders would soon be overtaken, while they became embroiled in a series of maddening patent lawsuits. Wilbur, possibly weakened by the stress, contracted typhoid fever and died on May 30, 1912, when he was just 45 years old.

Source: How The World Discovered The Wright Brothers, Forbes Magazine, Dan Ackman, nov. 18 2003

- Innovation costs time and money.
- When you're done, your idea is vulnerable: anyone can use it.
- Patents are supposed to protect you,
- but the system is imperfect.
- Once the idea is out, everybody profits.

## R&D considerations

With this in mind, observe the determinants of R&D expenditure:

- How large is the advantage that an invention will give? Is it easily copied?
- How large is the market in which the invention can be sold?
- How long can you keep a monopoly position in the market, before another invention comes along or the patent runs out?
- What is the probability distribution of R&D outcomes?

Notice that it is promising patent protection *ex ante* maximizes growth, but not delivering it *ex post* maximizes welfare.

# Joseph Schumpeter

Schumpeter (1934) saw the capitalist system as inherently cyclic.

- Technological progress drives growth
- Progress is caused by the desire to break monopolies
- Creative Destruction



Source:

<http://www.cpm.ehime-u.ac.jp>

Schumpeter's view:

- Research can cause a breakthrough that lowers costs
- The inventor can use this to establish a monopoly and earn rents
- Other inventors are busy creating something even better
- After a while, a new monopolist takes the place of the old one

## Research and Monopoly power

Notice that in Schumpeter's world:

- Monopolies are good
- Recessions have a useful role

This theory was hard to formulate in Schumpeter's day. However, it was the inspiration for many endogenous growth theories of the 1990s.

Schumpeter policy advice

Technological change is the result of profit seeking and should be encouraged.

- Only crack down on monopolies if they are 'unfair'.
- Legal system should protect entrepreneurs.
- Structural change is good: weak labor protection

## One-country model

So if technology becomes better with R&D, what can we say about growth?

- Simplify production function: each worker produces  $A$ .
- On aggregate, remove uncertainty in discoveries: each researcher increases  $A$  at a rate  $A/\mu$ .
- Divide people up in producers and researchers:

$$L = L_Y + L_A = (1 - \gamma_A)L + \gamma_A L$$

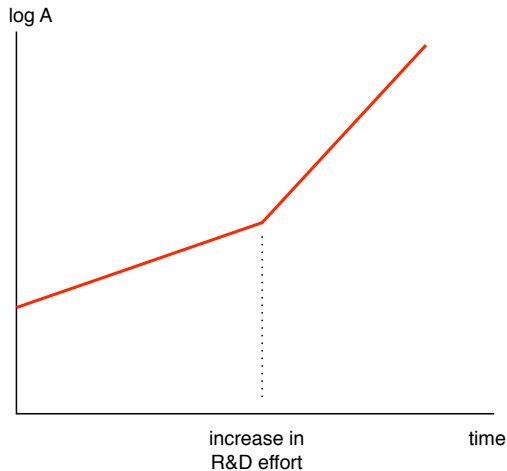
This gives technological progress as

$$\hat{A} = \frac{\dot{A}}{A} = \frac{\gamma_A L}{\mu}.$$

Notice: this is a model of everlasting growth; it is also an endogenous growth model in that it explains the rate of growth as an endogenous variable.

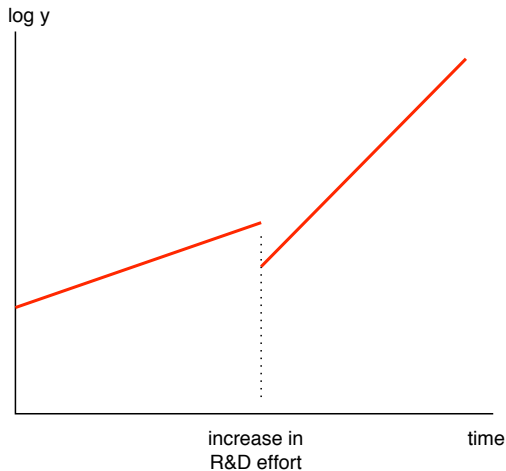
# A change in $\gamma_A$

Suppose the number of researchers increases; what happens to  $A$ ?



## A change in $\gamma_A$

Suppose the number of researchers increases; what happens to  $y$ ?



## Solow and technological progress

Remember when we relaxed  $n = 0$  earlier:

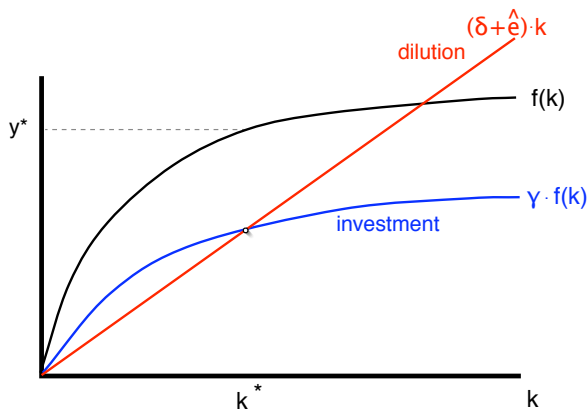
- Population growth acts just like depreciation:
  - depreciation: a fraction of the capital stock disappears:  $K/L$  falls.
  - population growth: an extra fraction of the population appears:  $K/L$  falls.
- Technological progress acts just like population growth:
  - Assume labor-augmenting progress:  $F(K, E \cdot L)$ .
  - Increases in  $E \cdot L$  can be caused by either factor.

Nice. So the graphical effect of population growth or technological progress is the same as an increase in the rate of depreciation.

But beware: the implications for productivity are different!

# Solow diagram

On the axes: capital per effective worker and production per effective worker.



## Rates of growth

According to the latest Solow model, changes in GDP can be caused by changes in

- Capital intensity: When  $k$  is unequal to (lower than) the steady state value, this causes productivity growth.
  - The further you are from  $k^*$ , the faster you grow
- Number of workers: Total production is  $y \cdot L$ . Even at the steady state, population growth causes production growth.
- Technological progress: Labor-augmenting technological progress raises production the same way the number of workers does; it also raises production per worker.

The latter is exogenous: exogenous growth model.

## Rates of growth (cond.)

Suppose  $k = k^*$  in effective worker terms: we are at the steady state.  $L$  grows at rate  $g_L$ ,  $E$  grows at rate  $g_E$ .

	rate of growth	rate of growth
$y/E$	0	0
$y$ (GDP per capita)	$g_E$	$g_E$
$Y$ (GDP)	$(1 + g_E)(1 + g_L) - 1$	$g_E + g_L$

Use  $(1 + g_E)(1 + g_L) - 1 = g_E + g_L - g_E \cdot g_L$ , when the rates of growth are small the last term  $\approx 0$

# International technology transfer

Weil talks about some of the things that may go wrong when one country wants to use the knowledge of another. This explains the enormous differences in productivity even though knowledge is, in principle, free to move around:

- Technological advances may be capital-biased.
- Tacit knowledge may be required to use advances.

# Refereneecs

- Mankiw, N.G., D. Romer and D. N. Weil (1992). "A Contribution to the Empirics of Economic Growth". The Quarterly Journal of Economics, Vol. 107, No. 2, pp.. 407-437.